# A ROCKET DRIVE <br> FOR 

## LONG RANGE BOMBERS

(Über ednen Raketenantirieb fïr Fernbomber)

by<br>E. SHnger and J. Bredt<br>Ainring, Aug'st 1944

## Deutsche Luftfahrtforschung

UM 3538

Translated by M. HAMERMESH RADIO RESEARCH LABORATORY

## 4. Note by the Publisher

Tomard the end of the last century a fem farsighted individuals becare thoroughly convinced that man could fly. Today their nasos are all bat forgotten but their technieal achiovements will endure for centuries.

Today mit are on the threshold of maned fight between the planete. Drs. Bugen Silngor and Irene Bredt are prontnent among the handful of pioneers whose dedicated efforts heve made poasible this vista.
nuber einen Raketenantrieb fur Fernbomber" is based on more than a decado of effort by the authors. The material is condensed. This report contains only about onemthird of the information which the authors had available at the time of writing; all of the nathematical derivations and much of the supporting and aupplementiary information were omittede

In apite of this fact, the report is, in effect, a definitive treatise. It catalogs nem probleans and outlines solutions to the more firportant ones. For years to come it will serve as a storehouse of vital concapts for the serious student of rocket science. For these reasons, its publication at thia time aeems marrented.

Since 1945, Dr. Irenc Bredt (now Sllnger-Bredt) and Dr. Engen Sanger have ilivod in Paris, where they are employed by the Argenel de liAeronnutique. Dr. Snager is also presidant of the International Astronatical Federation.

Thile the Technical Information Branch, BUAER, Navy Department, has very generously furnished copies of their translation of wber einen Raketenantrieb fur Fersbomber" to many public libraries and research institutions, this is the flrat time the repert has been available for prolic asie. The publisier would like to thank the $\mathrm{J}_{3}$ S. Navy, without whose permission this publication would not have been possible.

Robert Cornog
Sapta Barbara, California
16 Novenber 1952

## FOREWORIS

The application of pure rocket propulsion to aeronautics suffers at present from linitatigns imposed on exhaust speed and flight velocity by constructional difficulties.

Because of the thermal stresses on the engines. the exhaust speed is not raised to the physically possible limits.

Because of the mechanical stresses on the airfranc, the velocity of flight has not yer gone beyond the velocity of sound.

On the basis of extersive physical and physico-chemical studies, we shall discuss some possibilities which are opened for the rocket propolsion of long-range military aircraft when these two limits are surpassed.

In addition several suggestions as to construction are made, which should facilitate overcoming the present limitations.

These investigations on the problem of long-range military rocket aircraft originated as a joint work of the two authors during the years 1937-1941 and were intended, together with the material of report LM -3509, to be a second valume of "Hocket. Flight Technique", by the senior author.

As a result of circumstances caused by the war, publication was postponed and the reaults of the work issued in abstract form in the present report.

Sänger
(sig.)
Bredt

Ainring (Upper Bavaria), August 1944
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## I. Fundamentals

The range of flight-speeds several times the velocicy of sound is the exclusive province of the pure rocket, which develops the propulsive jet entirely from the fuel carried on board the aircraft. The pure rocket can also compete in cost at lower speeds, it propulsive forces of great magnitude or short duration are required, or if no surrounding air is available, e.p. under water or outside the perceptible atmosphere of the earth. These special characteristics give rocket propulsion a broad domain of application to military techniques, which can be out lined as follows:

Propulsion of projectiles or bombs, in which the relatively strong; stiort duration propulsive forces can be achieved in most cases by powder-rockets.

Auxiliary drive for propeller-, or jet.aircaft, with operating persods generally under a minute, for which liquid rockets wich compression-drive can be used.

Auxiliary or principal propulsion for vessels with period of operation of several minutes, so that rocket motors having fuel pumps, but without high exhaust speed, ape required.

Main drive of aerial torpedoes against latd, sea, or air targets, with moderate to long times of operation, in which high exhaust speeds are inportant only for quite larre ranges.

Main drive of fighter or bomber aircraft, e.e. for fighter defense at very hieh altitudes or for military aircraft operating over very great distances. Both propulsion-time and exhaust speed set extreme requirements for the rocket motor. The last-mentioned applicarion, the rocket bomber, is treated in more detail in the present report.

Pure-rocket enginies make only very incomplete use of the energy made available by the fuel. However since the craft is not loaded down by the etiergy carried on board but rather by the weight of the fuel, this disadvantage can be counteracted by use of fuels with the maximum pos. sible energy content per unit weight. Thus rocket fuels represent, on the one hand, carriers of energy with maximum concentration of energy fer unit mass and per unit tank space; on the other hand, they are che carriers of those masses from which the jet of the engine is developed.

According to the method of feeding the fuel (which, in the tank, is liquid or solid) into the conbustion chamber of the rochet, we can distinguish between various modes of oferation of the rocket motor; e.g. rockets with periodic propulsion, which are characterized by moderate values of the work for feeding the fuel, the cemperature stresses in the walls in contact with the flame, the exhaust speed and the thrust; and rockets with continuous propulsion, with arbitratily high constant flame-pressures, high constant extaust speed, maximum thrust for given dimensions and maximum thernal stresses of the furnace walls.

The type of construction of the walls in contact with the flame is determined by these stresses.

The type of construction using the heat caparity of the wall-material gives very simple solutions, which are however applicable only to periodic propulsion, or to continuous propulsion over short periods at moderate furnace temperatures. For example, the 20 mm . thick metal wall of the jet-throat of powder-rocket at $2800^{\circ} \mathrm{K}$ and having thermal conductivity 4000 k cal/m $\mathrm{m}^{2} \mathrm{~h}^{\circ}$, begins to melt on the side in contact with the flame after 2. 4, B, 10 , 14 or 90 sec , if it is made of $\mathrm{Al}, \mathrm{Ag}, \mathrm{Gu}, \mathrm{Fe}$ or $\mathrm{Ni}, \mathrm{Pt}$, or $\mathrm{Ir}_{\mathrm{r}}$ resp; this can be shown by calcuiation and can be qual. itativejchecked by tests on welding torches.

Designs of combustion chamber walls using the best refractory materiais give somewhat more complicated arrangements and longer propulsion times, which are in general limited mainly by chemical changes in the wall material. The best heat resistant materials, (melting points given in ${ }^{\circ} \mathrm{C}$ ) which would be of interest in this connection are for example: beryllium oxide ( 25,00 ), molybdenum (2600), zirconium oxide (2700), magnesium oxide (2800), tharium oxide (3050), titanium carbide ( 3140 ), rhenium (3170), tungsten (3380), ztreonium carbide (3500), tantalum and hafnium carbide ( 3700 ) and graphite (4000). With these materials, using non-stationary the mal conditiops, the driving times can be extended further than the values giver previously.

Design using condenser jackets around the combustion-chamber walls is similar to that used in internal combnstion engines for controlling the hot, strongly superheated conbustion gases; it is however limited to moderate combustion temperatures and pressures for which the heat flow through tho wall is overywhere less than $\& \mathrm{~h}$. P/ cin ${ }^{2}$ so that the velocity of the coolant need not be raised above about $10 \mathrm{~m} / \mathrm{sec}$.

$$
L=4.5 \frac{B \pi}{i^{2} \sin }
$$

4

Firewall construction using forced circulation of the refrigerant in channels, of preferably one-dimensional extent, which cover practically without gaps all the wall surfaces couched by the flame, gives the possibilicy of controling also those high heat transfers through the chamber walls which occur unavoidably in using high-grade rocket fuels in uniform-pressure rockets, and which go far above $1 \mathrm{~h} . \mathrm{p} / \mathrm{cm}^{2}$, and can be even $10 \mathrm{~h} . \mathrm{p} / \mathrm{cq}^{2}$ or more in the jet throat. This type of firewall for rocket motor construction is used in the designs of the present work.

Aside from design of firewalls, the supply of fuel to the combustion chamber is, for uniform-pressure rockets, a special problem for the solution of which various methods have been used.

Placing the whole fuel supply in the combustion chamber has proved suitable in shortperiod powder rockets. Pressure tank feed of liquid fuel, because of the considerable weights of the tanks and comprassed air, is possible only for moderate driving periods and fire gas pressures. Fuel supply through gas-pressure pumps limits the tank size and gives longer driving pericds at moderate flame pressures. Fuel supply with ordinary pumps and turbine drive require special propellants or exhaust gas removal from the combustion chamber and results in increased fuel requirement per unit monentum; nevertheless, it does give high driving periods and flame pressures. Fuel supply with ordinary pumps using a turbine driven by steam from the refrigerant, where the ateam for the turbine is developed by vaporizing the coolant in the canals of the chamber walls and fire-jet, limits neither driving period, driving pressure, or flame temperature, and permits the use of the greatest exhaust speeds. This method is the basis of the rocket motors fescribed here.

Finally, one of the most essential construction problems for uniform pressure rocket motors is the choice of furnace pressure. The high-pressure rockets with furnace pressures above 50 atm. (which are necessary because of the high exhaust speeds required), are in practise driven up to 100 atm . They have small dimensions per unit thrust and are especially valuable combined with highest grade fuels, where the already high exhaust velocity can be increased by $22 \%$ through a furnace pressure increase from 10 to 100 atm., and by $\% \%$ through a change from 50 to 100 atm. Its doman of application is therefore especially that of rocket flight, e.g. for rocket bombers, where the requirement of high exhaust speed is most stringent. The high requirements on the fuel feed systen are no trouble when they are taken care of by the coolant-steam turbine mentioned above, which uses the heat from the forced cooling of the furnace. As a result of inereased gas-density,-velocity,-temperature, and - radiation, the specific heatflow from the flame through the furnace walls rises proportionally with the furnace pressure. This has as consequence the decisive difficulty that the protection of the walls in contact with the flame becomes more critical as the furnace pressure increases, since the heat transfer from furnace wall to coalant only increases as the 0.4 power of the coolant pressure, so that a practical limit of furnece pressure is reached at about lop atm,


#### Abstract

Similar general considerations apply to the air-frame. To the fundanental question, whether explosive propulsion by rockets over large distances shall be used with wingless, unmanned rocket-torpedos or with winged and man carrying rocket aircraft, it nay be said that for the "returning" aircraft, the range of use and the total destructive energy brought to the target (weight of bomb $\neq$ energy of explosive) is as large as for the rocket torpedo for equal initial flight speed, so that the conserving of the empty craft for repeated use and the probably greater bombing aceuracy speak in favor of the aircraft. Since the initial cost of the empty craft is far greater than that for the bomb and fuel load, this is the basis for the choice. If che rocket bomber doesn't return to its place of takeoff, its range for equal $v_{0}$ will be much greater than that of the rocket torpedo, though, of sourse, the \% weight of deptructive energy brought to the target decreases. The extreme ranges possible with the rocket bomber are completely forbiden to the rocket torpedo.


The rocket bomber will differ from the present-day propeller-driven bombing aircraft in the following essential points: in place of the propeller propulsion from the fuselage iront it has the rocket propulsion in the fuselage stern; the fuselage is in the shape of a bullet with tapered hind part, the wings have a thin wedge-shaped profile with sharp leading and trailing edges and high wing loading at the start of the flight; the cabin is constructed as an airtight stratosphere chanber.

For starting, the use of its own fuel as in the usual propeller-driven aircraft was conkidered. Because of the great difference in start- and landing-weight this leads to large wing purfaces and too high fuel consumption in the range of speeds below the relocity of sound. Vertical start under its own power has only the last disadvantage, but even in a greater degree. Sling-starting on a horizontal take-off path until the sound velocity is reached appears most fovorable and is assumed here. In this type of start, by means of external forces, an especially
energy-consuming part of the aircraft acceleration is not obtained at the expense of the fuel carried on board, so that the range of the ameraft catapulted in this fashion rises markedly, Wile at tide sume crme the flighr-characteristies can be matched more easily to che steadily decreasing wing loading during. flight.

As methods of flight were considered; acceleration to the point where flight speed equals exhaust speed, and then flight at constant speed; also acceleration to such a speed that the aubsequent unpowered glide extends over the entire remaining flight path. For equal fuel cost, the last method gives greater ranges and simpler power plant, and is therefore assuned from now Du.

The whole procedure for use takes place sonewhat as follows: the rocket bomber at the burface of the earth is brought to a speed of about $500 \mathrm{~m} / \mathrm{sec}$, by a ground-fixed rocket drive in a period of 11 seconds over a 3 km , etarting path; then climbs at full motor drive to a height of $50-150 \mathrm{~km}$ along a path which is inclined at $30^{\circ}$ to the horizon at first, but later becones flater; thus it reaches final velocities up to more than twice the exhast speed. The duration of the climb is $4-8$ mirutes; usually during this time all the fuel supply on board will be consumed. At the end of the climb the rocket motor is turned off, and the aircraft, because of its kinetic and potential energy, continues on its path in a sort of oscillating gliding flight with steadily decreasing amplitude of oscillation. This type of motion is similar to the path of a long-range projectile which from similar heights foilows a descending glide-path. Because of its wings the aircraft descending its ballistic curve bounces on the lower layers of the atmos phere and is again kicked upwards, like a flat stone ricocheting on a water surface, though during the entrance into the dense air each time a fraction of the kinetic energy is consured, so that the initially big jumps steadily become smaller and finally go over into a steady gliding flight. At the same time the-flight speed, along the glide path of several thousand kilometers, decreases from its high initial value to normal landing speed. If the descending path (which is within certain limits controllable by the pilot) lies in the direction of the target, the bombs are released at a predetermined moment, and the craft returns to its starting place (or some othan landing field) in a wide arc, while the bombs go toward the target along the original direction of flight. Even if the target is very distant frou the take off point, the bombs are only dropped near it, so that the scatter of bombs can be compensated for by a large number of releases on the target, which will in this way be covered by a Gaussian distribution of hits. This military use is completely indeperdent of weather and time of day at the target, and of enemy counteraction, because of the possibility of using astronomical navigation in the stratosphere and becadise of the height and speed of flight.

From the characteristics given for the rocket bomber it follows that this is not the development of an improved military craft, which will gradually replace present types, but rather that a problem has been solved for which no solution existed up to now, namely, bombardment and
.- $=$ bombing over distances of 1,000 to $20,000 \mathrm{~km}$. With a single rocket bomber point attacks can be made, e.r. from Central Europe, on distant point targets like a warship on the high seas, a canal lock; even a single man in the other hemisphere con be fired upon.

With a proup of 100 rocket bombers, surfaces of the size of a large city at arbitrary places on the earth's surface can be completely destroyed in a few days.

## 1. Characteristics of the Rocket Motor

The min parts of the basic constraction of the rocket moter considered here are athom in Fig, 1. The fuel goes from the fuel tank to the fuel pump, where it is compressed to 150 atme, then fed continuoualy through valve 5 to the anjection head of the combastion chamer. The oxyger goes from the thin-walled uninsulated wygen tank into the oxygen talk, is compreased to 150 atm. there, then foreed through ralve 6 and the pipe syatem of the condensera into tha injoction head and the conbustion chamer, after being warmed to $0^{\circ} \mathrm{C}$. There the fuela cone together for the first time, mix and burn producing flue gases at a constant pressure of 100 atm. and at $4000^{\circ} \mathrm{C}$. $>$ In the head of the burner, the five gas expands to a very low pressure and form the driving masajet with exhaust velocity of $3-4000 \mathrm{~m} / \mathrm{set}$, whose reaction for a fuel consumption of $245-327 \mathrm{hg} / \mathrm{sec}$ produces a thirust of 100 tons. With a 90 ton ${ }^{\text {fuel }}$ supply, the aircraft can be driven with the nbove thrust for 367-275 seconds.

Aside from this min process with energy conversion of about a million Kcal/sec the necondary process shown schematically in Fig. 2 gives an energy conversion of 20,000 Kcal/aec for driving the feed-pumps. Its partay visible in Fig l, can be fallcwed from the witer puap which puts about $28 \mathrm{~kg} / \mathrm{sec}$ of water under 250 atm. gressure, drives this water, the jet thront, intc the cooling pipes of the jet walls, where it flows foward the mouth of the jet and is heated to shout $300^{\circ} \mathrm{C}$. After coming out of there, while still above the critical pressure, it is (agaln at the jet throat) driven into the cooling pipes of the conbumtion chanber wall, where it is again heated and vaporizes in the neigtborhood of the critical pressure; finally it is removed at the injecsion head in the form of highly-compressed superteated steam, led to the steam turbina; therpe it expands to about 6 atm . and goes to the liquid-oxygen-cooled condensera where it is reconverted co water and gives up considerable energy to the oxygen; then it repeats. its cycle going through the water pump. The steam turbine drives all three pumpe from the sane shaft. During the process valves $3,4,5$, and 6 are open, and 1,2 are closed, while 7 serves as anfety valve against toc high rotation of the turbine.

The process described can be begun with the aid of the steam-starter, which produces the amall amounts of steam required by chemical mexnis in this procers the valves 3 and 4 are ciosed, 1, 2, 5, 6, and 7 are opened.

Aside from the detaila given in the literature, ( $16-30$ ) the following things are important for understanding the proposed construction:

The relative value of different fire-wall materials is determined by the avalable beat flow $q=\lambda / t_{f}-t_{4} / / / 2$ through the walls for a given heat capacity of the wallit. If the wall thickpess $d$ is proportional to the reciprocal of the breaking strength $\sigma$ (tensile stress), or to its square root (torsion stress), then the possible heat flow (and also therefore the worth of the material) is proportional to the product $\lambda\left(t_{f}-t_{f} / \sigma_{0}\right.$, or $\lambda\left(t_{f}-t_{f} / / \sigma^{-}\right.$resp. Here $\lambda, t_{f}$ and $\sigma$ ore pure material constants, while the cool-surface temperature $t_{H}$ (and so the whole evaluation) depends on the particular arrangement, coolant temperature, etc. In the conbustion chamber cooled by live steam, the cool-surface temperatures are $500-600^{\circ} \mathrm{C}$. From Fig. 3 one sees that the ual beat-resistant metals chrome-nickel steel, nickel, "ventil"-steel, etc., are favorable (as confirmed by construction experience) while platinum is even more suitable. The theoretically most favorable materials like tantalum, tungsten, and molybdenumare, because of cheir chemical activity and the difficulty in working them, actually not at all promising. In the hot-water-cooled fire jet the cool-surface teaperatures are at $400+500^{\circ} \mathrm{C}$, because of the high heat flow; Fig. 4 shows (in accord with practical experience) that copper is unsurpassed as jet-wall materiai.

The cooling syatem for the walls in contact with the flame (21) is required because of the high heat flow from the conbustion gases to the fire -walls; this is presented in Fig. 5 for a gas-oil-oxygen charge at 100 atm. contustion pressure, on the basis of talculations and practical experience. For example, in order to conduct 5 hp ./ gip ${ }^{2}$ from the fire-surface to the cool surface through a 1 mm thick copper wall, a temperature gradient ( $\epsilon_{f}-t_{h}$ ) between the two surfaces of $100^{\circ} \mathrm{C}$ is necessary. In order for such thin walls to withstand the mechanical stresses due to flame-and coolant-pressures, they must be reinforced at very short-intervals. At the same time the heat flow through the fire-walls must be assured by a precisely prescribed and carefully maintained high streaming velocity of the coolant behith the fire-wall. Both requirements can be met by the cooling-pipe system shown in Fig. 1, with foreed circulation of the coolant in structures of onedimensional extent. The necessary compromise between the rising forced motion and increased pressures required to drive the coolant when the number of channels is decreased has been so made that the cooling system of the jet consists of several hundred parallel pipes each only a few


Fig. i: Systematic representation of the main parts of the rocket mot of the rocket bomber.

or, drawn in the interior


Fig. 2: Energy flow in the morking process of the rockot motor.
sq. mm . in eross-section, which join together partially at the jet throat and again branch off in the far parts of the jet mouth so that the small individual cross-section of tire pipes is retained; each pipe rums meridioually, and the whale surface of the jet is completely covered with pipes, of which the side toward the flanc consists of the smooth and torsion-stressed firewe 11. The requirement of a plane surface does not apply to the furnace surface in contict with the flame, so that there the statically more favorable circular cross-section can be used. This circumstance and the smaller heat-flow through the furnace walls require gfeater wall-strengths of the firewall and thus greater cross-section for the individual cooling pipes, so that the then necessarily few pipes in parallel can assure high circulation of the coolant in the neighborhood of the steam chamber which has varied and unstable flow conditions. These few cooling pipes are wound on the furnace surface in the form of an evaporating coil.

We see, from the above discussion, the requarements on the coolant itself: large heat capacity, beat conductivity and density; for this reason mercury has advantages over water.

The reason why the furnace-jet in Figs. 1, and 5 is shown with the unusually large opening angle of $60^{\circ}$ is the following: (22) Aside from the fact that special coolant, which best suits the requirements, is circulated around the fire-walls, the actual coolant is really the fuel itself, to which the intermediate coolant transfers its heat in the pumps and condensor. The heat-absorptivity of the fuel before it is brought into the combustion chamber is limited, and is only a few percent of the heat which is liberated when it is burned in the combustion chamer. One must therefore take care, that the total heat transfer per sec. from the flue-gas to the coolant through the walls of the furnace and jet, which is given as $20 \%$ in the example of Fig, 2, remains leas than or equal to the aforementioned absorptivity of the fuel consumed per sec. This total heat transfer which must be regulated is proportional to the total inner surface of the furnace and jet. It can be decreased by diminishing this total surface. Concerning the contribution of furnace and jet to the total surface in contact with the flame, the following is true: all experience shows that over a wide rante of values, combustion in a furnace is more complete, and efficiency and exhaust speed are correspondingly larger, the greater the furnace volume Vo as compared to the smallest cross-section $f$ of the fornace jet. Because the total wallsurface is limited by the heat absorptivity of the fuel used as a coolant, the furnace surface can be increased provided that the jet surface is decreased keeping the sun of the two below the pernissible limit. From Figs. 2 and 5, the total heatflow through the 154,000 em of fire-wall surface represents $2 \%$ of the energy developed, which corresponds to the permissible theat absorption of the fuel, so that the heatflow per unit area of the furnace and jet walls is about 0.8 nif, $\mathrm{cm}^{2}$. If in place of the short $60^{\circ}$ throat with $60,000 \mathrm{~cm}^{2}$ surface we used the customary lavalthroat with a $10^{\circ}$ opening angle, its furface ( $345000 \mathrm{~cm}^{2}$ ) conld not be completely cooled if the same heatflow and absorptivity of the coolant were maintained; for the furnace there would be no surface cooling available at all. At the same time the length of the Laval-throat could not be decreased below 9720 mm . By using large throat-angles we are enapled to fulfil the requirements of the fuel-cooled rocket motor, and morecver the quantity Vo/f' which determines the completeness of combustion can be increased, so long as the increased efficiency of the furnace $\bar{\sigma} / E$ is not overcome by the decrease of jet-efficiency $\left(A c^{2} / 2 \mathrm{~g}\right) / \sqrt{0}$ with increasing opening angle.

The pump system of the rocket motor consists of three pumps for fuel, oxygen and coolint, and the driving turbine, for these purnps. The vaporized coolant of the rocket motor is used for feeding the turbine, where the forced circulation used for the combustion chamer permits the chamber to be used a high-pressurc-radiating-steamboiler with forced circulation in the manner of the Hens on -, lahmont-, lelox -, Sulzer - stean boilers. (27) Tite use of the vapormedi coolant for driving the auxiliary turbine has the advantage over the use of separate energy source that the total fuel consumption per inpulse by the rocket motor is not increased by the auxiliary turbine drive; the advantages over feeding the auxiliary turbine from the flue-gas of the rocket motor are that: use of the cooling capacity of the fuel for cooling the combustion gases brings it to temperatures permissible for the turbine drive-pod; the difficulties associated with condensation of, for example, metallic-oxide fumes in the flue gas disappear; the heat absorptivity of the fuel as a coolant is increased by the work done in puniping the important decrease in nomentum of the jet in che emission of a part of the flue-gas and transfer of its heat content to the remaining flue-gas is avoided; finally the construction of a high pressure steam turbine is incomparably simpler than that of a high pressure flue-gas turbine. Since accerding to Fig. 2, the efficiency of the 12,000 mp. drive of the steam turbine, which uses waste energy, iunimportant, while we do demand very small weights of the installation, the simple Curtis-shaft gives a suitable solution. The 3 pumps can in $v$ jew of the hagh total fuel supply of over 1000 $\mathrm{m}^{3} / \mathrm{br}$. , be designed as one-stage turhines (despite the high intake pressures), so that the whole pump assembly includint the turbines consiste of 4 running fron the eame shaft, at about 12.060 PHA. Thus the outer dimensions and weipht of the whole installation can be kept below $600 \times 1200$ am and 50 Gkg .


Fig. 3: Relative sualuation number of combusticn chamber wall materials subject to tension.


Fig. 4: Relative evaluation numbers of firewall construction materials under bending (exhaust wall materials).


Fig. 5: Conditions in a 100 ton rocket motor operating with oxygen and oil.

Aside from the requirements of extremely light construction the turbine, fuel pomp and water pump present no special constructional difficulties, whereas in the construction of the oxygen punp the choice of the construction material, the arrangement of the moving parts and the feeding of the bofling liquid to the pump must be specially considered. As a construction material (for the oxygen pump) which at $-180^{\circ} \mathrm{C}$ will he sufficiently strong, elastic and resistant to impact, sufficiently resistant to corrosion and non-inflamable in liquid oxygen, the nickel-, Al-, and $\mathrm{H}_{\mathrm{n}}$ - bronzes, as well as Monel-type alloys and pure nickel have proven satisfactory. In view of the inflamability of all lubricants in liquid oxygen, the problem of arrangement of moving parts was solved by using a floating support for the pump shaft away from the oxygen. In order to drive the boiling liquid $\mathrm{O}_{2}$ from the tank steadily to the pump, an arrangement was used vhereby the oxygen flows toward the pump over a long route in the direction of an acceleration field; A.g., in the test installation, from a higher level; or in the aircraft, from tanks lying far toward the front. Because of the gradual pressure increase in the feed lines, accompanied by only a slight temperature increase, supercooling of the $\mathrm{O}_{2}$ occurs at the pump intake, so that no more gas is liberated.

Fig. 6 is a photo of the experimental model of a high-pressure liquid- $G_{2}$ pump, which as a rotary 6 -stage pump with external bearings supplies $5 \mathrm{~kg} / \mathrm{sec}$ of liquid $\mathrm{O}_{2}$ at 150 atm . pressure, when running at 15,000 RPNI; it has proven its suitability and reliability in hundreds of experiments.

The ignition of the rocket motor is not shown in Fig. l, because ignition is inited only to starting; once the combustion chamber gets going it operates like a welding-burner. The basic ignition procedure chosen was the injection into the combustion chamber of materials which ignite on contact with $O_{3}$ or air. From the pyrophors to be considered, like the phosphorus hydrides, "silanen", halogen-acecylenes, rare earth amalgams, metal alkyls, etc., zinc-diethyl $\mathrm{Zn}^{\left(\mathrm{C}_{2} \mathrm{H}_{5}\right)_{2}}$ was choser at the suggestion of H. Troitzach; F. Zohrer developed a suitable ignition fluid by dilution of this with heavy hydrocarbons (e.g., machine oil), and also an ignition apparatus in the form of a small pressure bomb using compressed nitrogen and a remote-contralled valve; by a simple movement of the valve and consequent, injection of the ignition fluid into the combustion chamber, arbitrary ignition time and arbitrary repetition of the ignition is possible. This ignition procedure is notable for its sure performance and the very smooth starting of combustion.

The practical work on the development of the rocket notar described in this section was taken up by the senior atthor in 1933-34 at the Technische Hochschule in Vienna and gave in the first experiments, on small models with 30 kg . thrusts, controllable flame-pressures of 50 atm. and high evhaust speeds; the fuel was (b) (at up to 150 atm . injection pressure), and gas oil (at up to 500 atm. injection pressure), and a Laval-throat of small opening angle was used, (19) After a delay of several years, which were spent in constructing larger experimental installations, the sests were reconmenced at the Trau Aeronautical Testing Station in 1939. The construction of the experimental installations was under the direction of H. Zborowski; the construction of the components was directed by H. Ziebland; K. Hedfeld directed the experimental work. Fig. 7 shows the testing-shed during an experinent with 1 ton thrust and 5 minutes duration. Among the important.parts, one can see at the left on the embankment a cylindrical tank of capacity $2.5 \mathrm{~m}^{3}$ for the liquid $\mathrm{O}_{2}$, and just to the right of it the tap for the underground tank of liquid $U_{2}$ (see also II, 2). The drive tank, from which the apparatus is directly fed, is an open uninsulated thin-walled metal tank which (out in the open) vaporizes oxygen at the rate of $15 \mathrm{~kg} / \mathrm{hr}$. per sq. meter of tank wall, and whose varying weight during the test is shown by an automatically-recording spring balance. Fron this tank the liquid $O_{2}$ flows at slow speed under its own weight to the high-pressure liquid oxygen puap 8 m . below (see fig.). Bevond the feed pump the liquid axygen, now at 150 atm , pressure, runs through a heat exchanger, in which it is heated by the warm cooling-water coning from the furnace, then goes into the combustion chamber through a large namber of injector nozzles. Following the corresponding path of the fuel, we see in the left foreground a $1 \mathrm{~m}^{3}$ fuel tank, from which the fuel flows under its own weight ta the high-pressure fuel pump. For this purpose a cog-wheel pump is used, which compresses the gas-oil to 150 atm . at 3000 RPI . In the experiment shown here, the fuel and oxygen pumps were driven together by a D.C. Notor standing between the pamps; later the coolant-steam turbine was used instead. Beyond the fuel pump the fuel also is foreed into the combustion chamber through a large number of nozzles. The fuel - and $0_{2}$ - streans are directed at $30^{\circ}$ to each other and have initiai entrance-velocities of about $100 \mathrm{~m} / \mathrm{sec}$ so that rapid spraying and mixing is forced. In the furnace, the three fluids - oxygen, fuel and ignitor - meet and form the furnace gas, The furnacegas pressure during the entire run is up to 100 cm , ith $V_{0} / \mathrm{fr} 800$ and about $30^{\circ}$ opening angle of the provisional expansion-nozzle. The next photos 8 and 9 show a 1 -ton trial frow the jet side, 10 shows a small model using coolant vaporization; Fig. 11 shows a l-ton trial in which a high-percent Al-gas-oil suspension was used as fuel. Thie flame glows brighter in this case, and the resulting aluminiunsoxide begins to condense to white corundum dust at a few meters from the jet opening, and then thickens into a heavy white cloud. Finally, Fig. 12 showis a ahort-exposure


[^0]

Fig. 7: Overall vien of a l-ton, high pressure combustion chamber experiment using cooling by evaporation. Propellent tanks are above roof to the left. The fuel purpps directly underneath. Combustion chamber is in oparation in center. Note the cloud of condensed cooling agent. The observation stand is above on the right.


Figare 8; Overall vien of a rocket motor test atand. This motor produced 1 ton of thrust for a duration of 5 minutes.


Figure 9: View of insuruwents and propellent lines. This test was run on the 20th of Karch 1941. The chsmier pressure 100 atmoapherea, the thrust 1.1 tons, the duration 3.5 minutes.


Figure 10; Small water cooled combustion chamber and test instrument in duration test. Water, heated at $400^{\circ}$ centigrade at 100 atmospheres pressure in the cooling gystem.


Figure 11; Rocket motor teat stand experiment using Alumintm in oil dispersion as fuel. Supersonie exhaust gases from the nozzle of l-ton experimental rocket motor. Note compression 1ines.
 compression lines.

photo of the jet itaelf, in whicn one can see whth the naked eye the appersonic compreasion linequ which aive the exhonat jet the appearance of a large, blue cryatal.

The min part of the practical work consisted in the congtruction of combustion chambers for developing and withistanding of conbustion gases with the high energy concentration mentioned fio time could as yat be devoted to the conversion of the heat into kinetic energy of a jet, that is to construction of a jet nozzle. Ordimary amoutce of fuel consumption in the course of long and steadily run tests, geve effective exhaust spedds of up to $2400 \mathrm{~m} / \mathrm{sec}$ at about 35 atm flame preanure, with gas-oil-oxygon fuel and for $k=f / f^{-1} 1.43$ (i.e., atill insufficient performance of the extended throat section).

With the relatively $10 w \pi$ walues of the conbustion gas (about 1.25 ) carefully constructed fire-jets should, accarding to Fig. 17, give ot least $P / P f^{\prime \prime}=1.6$ in the test set-up, and about
 hataf apeeds of $2700 \mathrm{~m} / \mathrm{sec}$. on the ground, and near $3000 \mathrm{~m} / \mathrm{sec}$. in the aircraft.

Figally, Fig. 12a shows a trial construction of the carbaretor of a 100 ton rocket high pressure combustion chamber, which was not hovever wed in the experiments.

## 2. Effective Exhast-speed of the Rocret Motor

Statenients conceraing the form and natire of the phyaica-chemical processes in the jet of the rocket-furnace, which could give a dererminatign of the effective exhaust speed, assume. : knowledge of the processes and of the final sate of the combuation gas in the furnace.

Ore may assume that most of the available tame in the furnace space (about 75 millisec) is used up in processes of spraying, heating, vaporization, diasociation, turbulence and diffusion of the injected streams of fuel and oxygen; and only a mall part of the time is used for the actual combustion and coming to equilibrimm. The fully propared and mixed fuol-, and oxygena molecules (or - atoms) callide, and react with each other, but will immediately dissociate again if there is no means of transferring the liberated heat of reaction to internal degrees of freedom, to other bodiea, or converting it to trabalational energy. The last possibility exista for atonic collisions (according to the principle of the conservation of the center mass) only if a third atom or molecule takes part in the collision, so that the particles present after the rraction can repel each other. (Triple collision, wall-catalysis, exchange reaction). A measur for the probability of occurrence of any reaction is the affective number of callisions, which sitates how many col lisions of other particles with the molecule under consideration are required to produce the desired effect. According to an empirical fornwla of Gerb (5) z tormeriple collisions oecur in every $Z$ ordinary collisions Aftar each formation of a new molecule, later collisions will supply energy first to its ratational and then to its vibrational degrees of freedom at the expense of its translational etiergy, till finally in some cases dissociation occurs. In the course of a aufficiently long thme, which is dependent on the number of effective collisions which a molecule must undergo for each of these changes, and on the time interval between two such collisions, an equilibrium state flependent on the pressure, temperature and propoftions of fuel and $\mathrm{O}_{2}$ ) develops in the furnace, which can be exactly described in terus of the kind, number and energy content of the moleculei or atons prosent. This state is aszumed in the later calculations of the effective exhanst speed.

Since the grentest possible energy content of the translational degrees of freedom of the combustion gas deteraines the maximum possible valite of the effective exhanst apeed, one would prefer for the rocket motor a more favorable final state than that of stationary equilibrium, this, seemp to be attainable, since translation, rotation, vibration, dissociation and recombination take successively longer times to attain equilibrium, and the time draing which the fuel remins in the furnace may lie anywhere between these times. According to Jost (8, page 141) it is conceivable that the nevly formed molecules ma, because of their process of formation, not hive their vibrational degrees of freedom completely excitied by the ond of the combustion pronebp, so that a greater fraction of the energy remains for the K.E. of the center of mass than corresponds to equilibrium; thus temperature and pressure at the end may be higher than that corresponding to equilibrium. (see K. Wohl and M. Magat, Z. Phys. Chem. Vol. 19, p. 117, 1932; also (10) p. 805, fig. 6, "The Approach to Thermal Equilibrium").

While, in the furnace of the rocket at say 100 atm. flame pressure and $3700^{\circ} \mathrm{K}$, a molecule experiences an average of $10^{\text {h }}$ callisions per sec, the number of collisions drops fter expansion in the jet, with a corresponding decrease in the rate of excitation of the degrees of freedom, the rates of further chemical reactions, such as reburning, and of physical reactions such as condersation or solidification of the combustion productes (provided the temperature drops below their atatic sublimation temperature dering the expansion).

The processes in the jet are then treated exactly as those in the furnace, neglecting all wall-effects; the expanding combustion gas is assumed to be an adiabatically closed system with c total energy E kcal/ kg corresponding to the beet content of the mixture; i.e. Apo

Accordingly as the time the streaming gas spends in the jet is long or short compared to the times of development of the various internal energetic and chemical equilibria, three assumplions are possible concerning the expansion:

1. The time spent. or the number of collisions which a molecule undergoes, on its path through the jet, is so small that no energy exchange or changes of vibrational and dissociation energy can occur. The characteristic conditions for this case are $00=0$ and $C$ osee $d=O$; when put into the generally valid energy relations between total energy, E, heat supplied, $Q$, internal energy, U, heat of dissociation, $D$, heat of vaporization at $0^{\circ} \mathrm{K}, \mathrm{F}_{\mathrm{io}}$, heat content, J, work done in expansion, Ap V, and kinetic energy $A=/ \ell_{\delta}$ so the equations:

$$
d U=c_{v} \text { trans } d T+c_{v i r a t} d T=-A p d V
$$

$$
D / Q_{0}=\left(T / T_{0}\right)^{\frac{x_{u}}{x_{4}-1}} \quad \text { where } \quad x_{u}=1+\frac{A R}{R_{1}\left(x_{0}+r_{0}\right)} \quad \text { is }
$$

a "smaller" adiabat exponent which results from only translational and rotational specific heats; ( 2 ordinarily would be (Cum $+A R$ )/Curate (note by Transl).

$$
\begin{aligned}
& \Delta J=\frac{x_{k}}{x_{m}-1} A R\left(T_{0}-T\right)^{\text {and }} \\
& \left.c^{2}=\frac{2 g}{A} \Delta J=2 g R \frac{x_{x}}{x_{4}-1}\left(T_{0}-T\right)=2 g R T_{0} \frac{x_{x}}{x_{x}-1} / 1-\left(D / \rho_{0}\right)^{\frac{x_{x}-1}{x_{u}}}\right]
\end{aligned}
$$

gives the variation of the flow -velocity as a function of the pressure drop in the jet.
2. If the time spent by the combustion gas in the jet is such that vibrational states are instantaneously in equilibrium, whereas no chemical processes can reach equilibria corresponding to the changed conditions, then the flow is characterized by $d \theta=0, C_{v}$ orc. $d T=f(T)$. From this and the fundamental equations we get the relation:

$$
\int_{0}^{\pi} c_{v o s c} d T+\left(c_{v} t_{\text {roans }}+c_{v} \cdot t\right) / d T=4 \int_{0}^{\pi} d V
$$

For a gas mixture with $n$ vibration frequencies and in different gases,

$$
\begin{aligned}
& \overline{C V}_{\text {otc }}=A R \sum_{i=r}^{n}\left[\frac{\left(0_{i} \delta\right)^{2} e}{\left(e^{2} \theta^{2}-1\right)^{2}} \cdot p_{i}\right] \text { and }
\end{aligned}
$$

$$
\begin{aligned}
& \text { also } \vec{X}_{A}=\left\lvert\,+\frac{A P}{E_{v}(t r o n s+r o t)}\right. \text {, where } p_{i} \text { is the canstont }
\end{aligned}
$$

partial pressure of the gas as compared to $p_{0}$ and $G_{i}$ the characteristic temperature. The internal for this type of flow is:

and

$$
\left.c^{2}=2 g R / \frac{x_{n}}{x_{0}-1}\left(\frac{\pi}{x}\right)+\sum_{i=1}^{n} p_{0} \theta_{i}\left(\frac{1}{N_{2}-1}-\frac{1}{e^{-1} / 1}\right)\right]
$$

3. The third possible type of flow in the jet occurs when the time spent in the jet by the combustion gas is so long that all energetic equilibria, including chemical equilibrium, can be attained for the instantaneous values of pressure and temperature. Thin type of flow is


84

From this and the fund mental equations, we obtain the differential equation:

$$
d\left(\int_{0}^{7} E d\right)+d D+A h d V=0 \quad \text { on } \quad d T+d D=A B T \frac{d p}{p}
$$

 cental analytic solution of the differential equation can be given. One aust therefore, using the difference equation
as close a collection of values of $J, D$, and $M$ as functions of $p$ and $T$ as possible, obtain the desired connections point for point. Procedures far calculating such tables for various fuels have been suggested by M. V. Stein.

The behavior of temperature $T$, hent content $J$, Dissociation energy $D$, and the liberated
 shown for the combustion of octane in liquid oxygen in Fig., 13.

Tho decision at to which of the three possibilities (or an intermediate one) is mont probable for the flow of the combustion gases from the rocket furnace is influenced by the following consideration: if the expansion takes place in an experimental short jet 1300 m . long, then the mean time spent by the combustion gases con be computed to be about $2 \times 10^{-3} \mathrm{sec}$; corresponding to an average no. of collisions of the molecule on its way through the jet of about $2 \times 10^{6}$. If one considers that $96 \%$ of these collisions already occur in the apace between the furnace and the smallest cross-aection, then one sees by comparison with the effective number of collisions ( $10^{2}-10^{\prime}$ ) for the various energy i terchanges, that we may expect equilibrium of rota lions, vibrations and possibly even dissociations for the processes occurring in the initial part of the jet, up to its amplest cross-section. Flow-type l becomes the more improbable the higher the furnace pressure and temperature for given velocity, the longer the jet, and the smaller the effective number of collisions of the combustion gas mixture. Therefore flow-type 2 is generally assumed in calculations on rockets; it gives closed integrable formulae. The actually rapidly varying $x^{c}$ can, for rough eatimetea, using the usual adiabatic flow formula, be replaced in first


This mean value is strictly valid only for an expansion to $T_{m}=0 \% \mathrm{~K}$ and $p_{\mathrm{I}}=0 \mathrm{~atm}$. If for example the month values are to be calculated for an expansion to $p=1 \mathrm{aEm}$, then two equations
 mouth pressure $p_{m} \neq 0 \quad$ For the example'gig octane combustion given above, we obtain in this mannev from the equations:

 fairly accurate values of $T_{m}$ and $c_{\text {m }}$ whereas all intermediate values between $T_{0}$ and $T_{m}$, as well as the corresponding $G_{n}$, are more inaccurate. Flow-type 3, in case its consideration can not be avoided, can be represented by the method of holier with the aid of entropy charts, which show $J+D+\mathrm{A}$ as ordinate and enable $c_{\mathrm{m}}$ and temperature values to be read off. Figure 14 shows such Mollier-diagram for a combustion gas of gas-oil-oxygen and gas-oil ozone respectively.

[^1]

Figure 13; Temperature T, Heat content J, Dissociation loss Dpt, and gain in kinetic energy plotted against the logarithm of the preseure paccording to the thrie possible streaming gradients in the nozzle.
(See text p. 24 \& FF). The gasses entering the nozzle are assumed
to be a completely burned stoichiometric mixture of the octane and
$0_{2}$ at pressure of 100 atmospheres.
Fhtropy J + Diasociation
energy D + Heat of evaporation A
8צ/โ80
2000


Figure '14; Entropy diagram after M, Fon Stein for the
Hozzle stream gradient and jil combustion.

## Test stand conditions

| $\boldsymbol{p}_{\mathrm{m}}=$ | 1 at | 1 at | 1 at |
| :--- | :--- | :--- | :--- |
| $\mathbf{T}_{\mathrm{m}}=$ | $1030^{\circ} \mathrm{K}$ | $1577^{a} \mathrm{~K}$ | $250 a^{\circ} \mathrm{K}$ |
| $\boldsymbol{c}_{\mathrm{m}}=$ | $2518 \mathrm{~m} / \mathrm{sec}$ | $2759 \mathrm{~m} / \mathrm{sec}$ | $3020 \mathrm{~m} / \mathrm{sec}$ |
| $\mathrm{c}=$ | $2654 \mathrm{~m} / \mathrm{sec}$ | $2949 \mathrm{~m} / \mathrm{sec}$ | $3260 \mathrm{~m} / \mathrm{sec}$ |

Gained energy in the mouth of the nozzle
in $\%$ of F . 29.3 35.1 42

Flight conditions in $d / d n=0.14:$

| $\mathrm{p}_{\mathrm{m}}=$ | $0,0729 \mathrm{at}$ | 0.1305 at | $0,250 \mathrm{at}$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{T}_{\mathrm{m}}=$ | $500^{\circ} \mathrm{K}$ | $1041^{\circ} \mathrm{K}$ | $2225^{\circ} \mathrm{K}$ |
| $\mathrm{c}_{\mathrm{m}}=$ | $2760 \mathrm{~m} / \mathrm{stc}$ | $3066 \mathrm{~m} / \mathrm{sec}$ | $3330 \mathrm{~m} / \mathrm{sec}$ |
| $c=$ | $2820 \mathrm{~m} / \mathrm{sec}$ | $3178 \mathrm{~m} / \mathrm{sec}$ | $3520 \mathrm{~m} / \mathrm{sec}$ |

Gained energy in the mouth of the nozzle
35.1
43.4

Optimum conditions in case of complete relaxation.
$\left.\begin{array}{l}p_{m} \rightarrow \\ T_{m \rightarrow} \rightarrow \\ c_{m}= \\ c_{\text {max }}= \\ c \quad=\end{array}\right\}$

0 at
$0^{\circ} \mathrm{K}$
$3496 \mathrm{~m} / \mathrm{sec}$
$2970 \mathrm{~m} / \mathrm{sec}$
-
Gained energy in the
in \% of E。
40.7
56.5 90.4

Completely lossless conversion of the total input energy E into kinetic energy would give a theoretical exhaust speed

$$
c_{\hbar}=\sqrt{2 g E / 4}=46.55 \mathrm{~m} / \mathrm{sec}
$$

The concepts of mouth velocity $c_{m}$, maximal velocity $c$ max, and effective exhaust speed c (19), require sore detailed explanation, In general, for rough calculations, standardediabaticflow formulas, using a fictitious average $\mathcal{K}$, whose magnitude and evaluation has been discussed already in presenting the three types of flow. Thus for expansion to a mouth pressure $\beta_{\mathrm{m}}$, the



$$
c_{m} c_{i m}=\sqrt{1-\left(\frac{n_{0}}{n_{1}}\right)^{\frac{x_{1}}{x}}=\sqrt{8-7 m / 70}}
$$

If re multiply the velocity $e_{\text {d }} b$ the mass blown out per second and f we obtain the mouthmomentum $\mathrm{J}_{\mathrm{m}}$ of the jet.

In experiments with rockers, if the moth pressure $p_{m}$ in the jet is equal to the pressure of the surrounding still air $\boldsymbol{p}_{\mathrm{a}}$ this mouth momentum is directly recorded by a dynamometer as the force $P^{*}$; it depends on the ext anal air pressure, ice. on the barometer reading, altitude of the testing-place, etc. and must not be confused with the thrust $P$, as can be seen from a consideracion of Fig. 15. There the cons tons between resistance and thrust for a flying apparatus and their values measured on he gr y are schematized.

By resistance to driven apparatus in flight we mean the vector sum of all the aerodynamic forces on the ae gated surfaces, Lat ${ }^{1 /}$ be this sum of all the pressures and frictional forces. In a wind tunnel or a tow test on a non-driven device one always measures a smaller
 jet. It follows therefore that $W^{\prime}=W^{\prime}+P^{\prime} f_{\text {in }}$. For the moving body, the sir pressure $p^{\prime}$ behind the


Figure 15; Relationsni"p between thrust and drag in flight compared to sea level test values.
$\%$
 The valun $P^{\prime}$ gust alway be included in rasiatance meaturesents on undriven devices, in order to


By thrust on an apparatus in flight, we man the vector sum of all the flave pressures on the wurface of the rocket in contact with flame (inger walls). In a test one alvays measarbz a smaller thrust $P^{\prime}=P$ - Anf where $\rho_{a}$ is again the pressure of still air. The thrust measured in ateat depend on the preasure of the surrounding air and for the effective thrust we obtain $p=p+p+{ }^{\prime}$

The effect of these two unavoidable additionn effects in a wind-tumnel or a test can be seen most clearly by calculating the resultant force (thrust-resistance) which acceleratea $a$ mass minc/8:

$$
m d y / f=P-N=P^{\prime}-W+V_{n}\left(p_{a}-p^{\prime}\right)
$$

For measurements on the ground, an unavoidable additional force proportiona $f$ to $f_{i}^{*}$ is thas included, which is different in measurements of ressistance and thrust, and for which special correction must be made. The correction is small at moderate velocities ( $\mathrm{e}, \mathrm{g}$. in the use of roclets for takeoff) and becomes largest for very high supersonic speedo (o.g. rocket projectile, long distance military rocket aircraft).

In order to calculate the effective thust $P$ of a rocket notor, one must add to the mouth impulse $\mathrm{J}=\mathrm{C}_{\mathrm{m}} \mathrm{dm}$ the total pressure of the combustion gas on the jet mouth ( $\boldsymbol{p}_{\mathrm{m}} \mathrm{f}_{\mathrm{m}}$ ): or one must add to the free dynamometer thrust in the tes:

$$
D^{\prime}=J_{m}+f_{m}\left(\rho_{m}-\rho_{0}\right)=C_{m} \frac{d_{m}}{d t}+f_{m}\left(\rho_{m}-\rho_{n}\right)
$$

the total thrust of the still air on a surfact the size of the jet mouth:

$$
D=D^{\prime}+A_{m} f_{m}=V_{m}+A_{m} f_{m}=c_{m} \text { dmat }+D_{m} t_{m}
$$

In the initially mentioned, most frequent ease $A_{p}=p_{m}$, mouth impulae $J_{m}$ and dynamometer reading $P^{\prime}$ are identical.

Thus the effective velocity $c$, which is independent of external air pressure, and which when wultiplied by $\frac{d x}{d t}$ gives the effective thrust is:


The determination of the effective exhaust speed $c$ has the following peculiar, technicalily
 $=1-7 / \%$ In the rocket motor the quantity which corresponds to this is the jet efficicncy:

$$
Z_{D}=c^{2} / c_{\text {max }}^{2}=\left(1-\pi / T_{0}\right)\left(1+\frac{x-1}{2 x} \frac{\pi / 10}{1-\pi} / \pi\right)^{2}
$$

(see also (19) p. 6). This expression implies that the higher temperatures and the quantities of heat at those temperatures are more "effective" than lower temperatures and quantities of heat at lower temperatures. This represents no contradiction to the energy theorem, since the effective exhaust speed is not identical with the actual velocity of flow of the combustion gas, but is larger than it. This relation has neverthe less a technical value, because the part of the initial heat content at lower (temp.) ranges can be made available only by special technical procedures (for gases, which soon tend to develop degeneracies, like condensation, it can't be done at all) and non-availability therefore leads to relatively smaller losses than one would expect according to the second law, for the non-available heat content.

Aside from the relations already given, we can, from the well known equation for the rate of ges flow through the jet throat

$$
\Delta G=y^{\prime} a^{\prime} f^{\prime} f^{\prime}\left(\frac{2}{x+1}\right)^{\frac{1}{x-1}} \sqrt{2 g \frac{x}{x+1}} 13 / i_{0}
$$

and the equation for the effective exhaust speed, obtain a frequently useful relation between $P$,

$$
\begin{aligned}
& f^{\prime} \text { and } p_{0} \text { in the form } P=k, p^{\prime} f^{\prime} \text { where } \\
& H=2\left(\frac{2}{x+1}\right) \frac{1}{x-1} \sqrt{\left.\frac{x^{2}}{x^{2}-1} / 1-\left(\frac{p_{m}}{p_{0}}\right)^{\frac{x}{x}}\right]}\left[1+\frac{x-1}{2 x} \frac{\left(p_{m} / n_{0}\right)^{\frac{x-1}{x}}}{1-\left(p_{0} / \beta_{0}\right)^{\frac{x-1}{x}}}\right]
\end{aligned}
$$

By means of this factor $k$, by which the effective thrust is greater than the product of furnace pressure and jet-throat surface, one can reduce the otherwise tedious determination of effective thrust of a rocket motor, for sufficiently lengthened jet, to a measurement of the furnace pressure and the outside pressure, which can be easily done with ordinary manometers. This type of thrust measurement is very convenient in trial setups as well as in measurements on aircraft in




Figure 17; Analysis of effective thrust of rocket motor with $x=1$.
flight. Conversely, in a test, if furnace pressure and effective thrust (or dynamomer thruat) are measured, the metmally effective $\mathbb{Z}$ can be determined.
 shom in Fig. 16 as functions of the ratio of furnace pressure to mouth pressure, for a frequently used value of $X=1.25$.

As is shown in Fig. 17 for the apecial case $\mathcal{K}=1.25$, the individual factors in the expres sion for $k$ csn be interpreted very intuitively; the total thrust $P$ of the rocket motor is made mp of the partial thrusts: $P^{\prime} \mu_{n} f^{\prime}$, arising from the pressure $p_{0}$ of the combustion gas on that spot, $f^{f}$, of the rear mali of the' furnace whith one sees when looking down the axis of the jet, throat. A cylindrical tube, closed at one end would show this zame effect; $P=\beta_{0} f=2\left(\frac{2}{x+1}\right)$ the increase relative to $P_{1}$, axises from the non-uniformity of the pressure distribution on the remaining furnace-wall aurfaces due to the pressure gradient in the direction of the jet opening

This increase relative to $P_{2}$ is caused by the pressure of the combustion gases igainst the extended jet.

The free thrust $P^{\prime}$ of the rocket motor is

and is show for comparison in Fig. 17 by the dotted curve.


A typical hydrocarbon reaction is the burning of octan in axygen:

$$
C_{8} H_{1}+12 \frac{1}{2} O_{2}=8 C O_{2}+9420+25874 c o / k g ; c_{4}=1655 \mathrm{~m} / \mathrm{cct}[11]
$$

The value for the upper limit of the heat of mixing is referred to $0^{\circ} \mathrm{K}$. From Fig. 18, it is clear that we must subtract from this value the $9.1 \%$ loss due to heat required for phyical separation (melting, vaporization) and that the pressure dependent losses due to chemical separa. tion (dissocistion) anount to $34.4 \%$ at 100 ath. furnace pressure, so that under these driving



Figure 18; Heat of evaporation, dissociation, enthalpy, firegas temperature, theoretical and maximum exchaust velocity and adiabatic gas exponent of the combustion products in the case of oxygen and octane with static equilibrium.
varying furnace pressures, as well as the later graphs and tables for other fuels, were calculated by M. V. Stein, taking account of all the dissociation products, and assuming that physical and chenical equilibrium are reacoud in the furnace, and that expansion can be followed, as a flow of the second kind, to an extfrinal pressure zero. In addition the lower limit of the heat developed by the mixture ( $E-\mathrm{H}_{\mathrm{i}}$ ), the fent content $\mathrm{J}_{\mathrm{p}}$. furnace temperature $\mathrm{T}_{\mathrm{o}}$, theoretical and maximum exhaust apeeds are plotted, as a function of the $\log$ of the furnace pressure, for the proper aver age $\approx$ corresponding to this expansion; also to give some ides of the range of variation of $\boldsymbol{\pi}$ for other typas of expaision and other jet-mouth pressures, the adiabat exponent corresponding to the instantareows furnace-state is shown as apper value, and the value of $\%$ for the calculations of flow-type 1 is shown as an upper limit. In this and the later graphs, which enable only a relative comparis on of different fuch, the effective exhatst speed $E$, which is the determining factor for flight, and whose value for octane combustion has been previously given, is not eapecially noted, It can be determined from the maximal velocity using the value of jet efficiency: $\nabla_{0}=\left(c / c_{m o x}\right)^{2}$

The expression for the total loss of the rocket motox is
numerically, for octane combustionat 100 atm. furnace pressure, 7 友 $=0.565 \times 0.25=0.465$ The flight is fixed in this and all the later calculations by $\mathrm{d} / \mathrm{d}_{\mathrm{m}}=0.14$, on the basis of the following consideration: in flight, the pressure behind the stern" of the rocket bomber drops
 100. The conbustion gas will actually spread over the whole $2.50 \mathrm{~m}^{2} \mathrm{~s}$ urface of the stern of the aircraft; i.e., the ratio of jet mouth surface $f_{\text {im }}$ to the jet throat surface $f^{\prime}$ will be about 50 ; throughout the working period of the rocket motor, corresponding to $\mathrm{d} / \mathrm{d} / \mathrm{d}=\sqrt{50}=0.14$.

In addition to the important and carefully studied hydrocathon-oxygen mixture, there is a second group of rocket fuels, having the comon property that they are elements in the first columns of the periodic table, and which when burnt in $\mathrm{O}_{2}$ give much higher energy concentrations per unit mass, and also gererally per unit volupe, than the hydrocarbona.

The following reactions were considered:
Untersucht wurden folgende Reaktionen:

| Be | $+0.5 \mathrm{O}_{2}$ | Beo | + $5930 \mathrm{kca} 1 / \mathrm{kg}$ | bei $0^{\circ} \mathrm{N}$ |
| :---: | :---: | :---: | :---: | :---: |
| 2 B | $+1.50$ | $=\mathrm{B}_{2} \mathrm{O}_{3}$ | + $4930 \mathrm{kcal} / \mathrm{kg}$ | * ${ }^{\text {a }}$ |
| 2 Li | $+0.5 \mathrm{O}_{2}$ | $=\mathrm{Li}_{2} \mathrm{O}$ | + $4750 \mathrm{kcel} / \mathrm{kg}$ |  |
| 2 Al | $+1.50_{2}$ | $=\mathrm{Al}_{2} \mathrm{O}_{3}$ | + $3920 \mathrm{kcal} / \mathrm{kg}$ | ** |
| $\mathrm{H}_{2}$ | $+0.50$ | $=\mathrm{H}_{2} \mathrm{O}$ | $+3900 \mathrm{kca} / / \mathrm{kg}$ | * * |
| M | $+0.50_{2}$ | $=\mathrm{MgO}$ | + $3330 \mathrm{lkca} 1 / \mathrm{kg}$ | " ${ }^{\prime \prime}$ |

Combustion of $A^{\ell}$ and $H_{2}$ are shown in Fig. 14 and 20 in the same form as for octane combustion. The following table gives the most important characteristics for these light-metalfuels with stochiometric liquid $0_{2}$ at 1,10 , and 100 atm. furnace pressure:

| Upoer vatue of heot of enitume ExAcollkil | BeO | $\mathrm{B}_{2} \mathrm{O}_{3}$ | $\mathrm{Li}_{2} \mathrm{O}$ | $\mathrm{Al}_{2} \mathrm{O}_{3}$ | $\mathrm{H}_{2} \mathrm{O}$ | Mind |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\searrow$ - | 5930 | 4930 | 4750 | 3920 | 3900 | 3330 |
| Heat of Wapurivation A: Chcalkol | 5040 | 2300 | 2780 | 1290 | 750 | 3750 |
| Weight of Solid Ligurd Phase in Candustion los at latm at 10 atm at 100 abm | $\begin{aligned} & 13.8 \\ & 18.0 \\ & 23.2 \end{aligned}$ | - | - | - | - | $\begin{aligned} & 43.0 \\ & 48.0 \\ & 54.5 \end{aligned}$ |
| Heat Content <br> Jin $Z$ of $E$ <br> atlath, at pato at rooats | $\left[\begin{array}{l} 20.6 \\ 23.8 \\ 28.4 \end{array}\right.$ | $\begin{aligned} & 40.5 \\ & 46.7 \\ & 51.4 \end{aligned}$ | $\begin{aligned} & 21.9 \\ & 24.1 \\ & 27.0 \end{aligned}$ | $\begin{aligned} & 21,5 \\ & 24.4 \\ & 28,1 \end{aligned}$ | $\begin{aligned} & 45.8 \\ & 51.7 \\ & 59.7 \end{aligned}$ | $\begin{aligned} & 28.7 \\ & 35.0 \\ & 44.1 \end{aligned}$ |
| Baiting Pint at lation | 3400 | 1990 | 1100 | 3250 | 373 | 3120 |


|  | BeO | $\mathrm{B}_{2} \mathrm{C}_{3}$ | $\mathrm{Li}_{2} \mathrm{O}$ | $\mathrm{Al}_{2} \mathrm{C}_{3}$ | $\mathrm{H}_{2} \mathrm{O}$ | ${ }_{8} \mathrm{O}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Furnacie Tampanoture | 3400 | 6210 | 2350 | 3700 | 2950 | 3350 |
| at 100 im | 3520 | 7010 | 2730 | 4070 | 3200 | 3970 |
| of 100atm | 4550 | 7680 | 3200 | 4700 | 3560 | 4850 |
| $x$ or $\bar{x}$ ut lotm | 1.24 | 1,125 | 1.320 | 1,280 | 1.300 | 1.11 |
| of 100tm | 1.22 | 1.110 | 1.315 | 1.260 | 1.250 | 1.10 |
| at 100atm | 1.20 | 1,100 | 1,310 | 1.240 | 1,220 | 1.08 |
| chth $[\mathrm{m} / \mathrm{sec} 7$ | 7050 | 6420 | 6310 | 5730 | 5720 | 5280 |
| $c_{\text {mox }}$ [nlsec 7 at latm | 3200 | 4090 | 2950 | 2660 | 3870 | 2830 |
| of 10 otom | 3440 | 4390 | 3100 | 2830 | 4110 | 3130 |
| at 1000tm | 3760 | 4610 | 3280 | 3030 | 4420 | 3510 |

In this calculation all heats of fusion were nesucted relative to beats of vaporitation; all possible dissociations of the end products of combustion were taken into acccunt.

B, hi, Al and $\mathrm{H}_{2}$ do not differ essentially from hydrocarbons in the ir behavior during combustion, but differ only in the numerical results; for example, li and Al, despite their higher heat production, reach lower maximal exhaut speeds than for hydrocarbons, because of the vaporization and dissociation losses. Be and hy are basically different. In the case of Be, the large heat of raporization of BeO pernits only a part of the burning masi to vaporize, while a larse part remains in the liquid state (fog) or solid state (dust), These parts can then make use of the upper limit value of the heating value of the mixture, so that despite the large heat of vaporization, high temperatures, and high heat content of the total mass occur, with consequent large values of $\mathrm{c}_{\text {gax }}$. These liquid and solid masses in the combustion ges have agreat effect on the expansion of cle cotal mass, which expresses itself in the form of a very small adiabatexponent: This $\bar{x}$ of the total mass is calculated from the weight fraction $k_{g}$ of the gas phese, the weight fraction $k_{f}$ and tpecific heat $c_{f}$ of the siquid-molid phase by the equation:

 same time represents the hoiling paint of BeO, since the conbustion temperature is determined by the boiling point of the Heb. For Np, whose lower heat value is actually negative, the large fraction of liquid-solid phase results in very high temperatures and heat concent of the cotal mass and especially low $\overline{\mathcal{X}^{2}}$ values, so that the burning mass rapidly loses the characteristics of * gas or vapor and approaches, the behavior of a hot lava.
F. on the given values of maximum exheust apeed, the rekultant effective exherat apeeds for the individual light-metal fuels at 100 atm. furnace pressure can be computed, if we take account of the jet efficiency corresponding to the $\alpha$ values for $d / d_{p}=0.14$, using the relation



In first approximation, $\boldsymbol{x}$ was obtinined from the somewhat too large mean value between To and o, instead of between To and $T_{m i}$ this leads to less favorable values.

(The approximate mean value for $X$ is 1.255 for octane and gives $c=3120 \mathrm{~m} / \mathrm{sec}$. For purposea of uniformity, these numbers were used in later comparison calculations).

For application to rocketa, in eddition to the effective energy concentration in the mass, the effective concentration $E_{v}$ in the tank volume is important, nince it determines the size of the fuel tanks, rate of feed of the injection pumps and the evaluation number $k$ of the fuel,


Figure 19; Heat of vaporization, dissociation, heat content, firegas temperature, theoretical and maximum axhaust velocity and adiabatic exponent of firegases for the burning of aluminum in oxygen with static equilibrium.


Figure 20; Heat of condensation, dissociation, enthalpy, firegas temperature, theoretical and maximum exhaust velocity and adiabatic exponent of firegases in the burnigg of $\mathrm{H}_{2}$ in $\mathrm{O}_{2}$ with static equilibrium.

| Berylliun sit Flüssigsamerstoff $\mathrm{E}_{\boldsymbol{V}}=1580 \mathrm{kcal} / \mathrm{dm}^{3} ; \mathrm{K}=1,23$; |  |  |  |
| :---: | :---: | :---: | :---: |
| Bor mit Flissigsaverstoff | 1640 | ; | 1.27; |
| Lithium nit Fltiss igsaumerstoff | 820 | : | 9.64 ; |
| Aluninium mit Flüsaigsaverstoff | 1390 | ; | 0.68; |
| Flingigwasserstoff mit Fliussigsaverstoff | 780 | ; | 1.22; |
| Magnesium mit Flussigsauerstoff | 340 | ; | 0.25. |
|  |  |  |  |

Thus we arrive at the result that, among the fuels in the second group only Be, B and liquid $\mathrm{H}_{2}$ are auperior to hydrocarbons. Be and $B$ are immediately eliminated since, under the conditions possible in the tank, they are in the solid state, so that feeding them into the high pressure furuace of the rocket motor is inpossible. If one should try to overcome this difficulty by wing the material in the form of wire or powder, then the concentration in the tank would be decreased 50 much that the calculated small superiority would be last. Use of fael in the liquid state is eliaiasted because of the high melting point. So apparently liquid $h$ is the only meterial in the second group of fuels which can compete with hydrocarbons. Its 22 s superiority is endangered by the fact that at the temperature of liquid $\mathrm{H}_{2}$, condensation of the surrounding air will start on the metal tanks in the airctaft, so that the rate of evaporation of the $H_{2}$ will be increased and the aerodynamic forces will be affected, unless special precancions are taken. Nevertheless, liquid $H_{2}$, because of its easy procurement and also for reasons to be discussed later, may be considered as a most promising rocket fuel.

Since, with this exception, the light metals are only slighty (if at all) auperior to hydrocarbons as fuel, a second possibility should be tested, - a combination of two e,g. in the form of metallic suspensions in mineral oils. Suspensions not only combine good feed characteristics (they are easily pumped) with high energy density (with consequent low requirements on tank volume and feeding speed); in many cases they have the amazing property that their heat content is greater than that of their individual components auch as hydrocarbons and light metala. (30) Fig. 21 shows cycles for Al- $0_{2}$ and octane $-\mathrm{O}_{2}$ at 100 atm. flame pressure. A comparison of these shows the interesting fact that the end temporature in the combustion of octane ( $3700^{\circ} \mathrm{K}$ ) is about $830^{\circ}$ lower than the boiling point of $\mathrm{Al}_{2} \mathrm{O}_{3}$. If one assumes that equal stbchionetric mixtures of octane $-\mathrm{O}_{2}$ and Al $-\mathrm{O}_{2}$ are burned together, that relaxion-free heat exchange takes place between all the molecules of both burning gases, and that finally the two burning massets do not interact appreciably chemically, then one sees from the two cycles that energy will flow from the higher temperature level of the $\mathrm{Al}_{2} \mathrm{O}_{3}$ to the colder octane. This will be heated at the expense of the heats of dissociation and vaporization of the aluminimm combustion gas until the boiling point of $\mathrm{Al}_{2} \mathrm{O}_{3}\left(4530^{\circ} \mathrm{K}\right)$ is reached. After this process of temperature equalization has occurred, the useful heat content of the octane will have risen considerably, while the heat content of the Al gas will not have changed appreciably; i.e., the heat content and maximum exhaust speed of this $70 \%$ Al-octane suspension are larger than for Al or actane alone. The advantagea of the combination are the following:

1. The temperature of the burning gases for the suspension has not taken on a value midway between those of the individual components, but rather the whole conbusting mass has reached the boiling point of $\mathrm{AL}_{2} \mathrm{O}_{3}$. The requisite energy has been obtained at the expense of the othervise unavailable eaergy of dissociation and vaporization of the $\mathrm{Al}_{2} \mathrm{O}_{3}$.
2. As a result of the temperature equalization between the two fuels in the suspension, the relatively slight dissociation of $\mathrm{Al}_{2} \mathrm{O}_{3}$ has been decreased, while that of the octane has increased markedly. Thus the average specific heat $f=J o / T o$ of the gas has increased, as one can see from the smaller slope of the expansion curve for the octane. Both these circumstances result in increased heat content $J=\leftrightharpoons_{p} T$.

One realizes, noreover, thot the random example of a $70 \%$ suspension chosen here, which has equal parts by weight of Al - and octane, combustion gases, may not give the best results for $C_{\text {max }}$ : The best Al - octane suspension will be one having enough Al to heat the entire mass to the boiling point of $\mathrm{Al}_{2} \mathrm{O}_{3}$. The upper heat values of the Al will be fully used in this case, since no metallic oxide raporizes. The extra heat of vaporization and dissaciation of $\mathrm{Al}_{2} \mathrm{O}_{3}$ are used to increase the heat content and dissociation of the octane. The best results are obtained for a $60.5 \% \mathrm{Al}$ - octure suspension.

The considerations concerning the Al - octane suspension can be extended to the other light metals. Fig. 22 shows for $\mathrm{Be}, \mathrm{B}, \mathrm{Li}, \mathrm{Al}$ and Mg suspensions with varying metal content, the chief


Metal content in the metal dispersion in parts by weight.

[^2]characteristics of the conbustion at 100 atm furnage preanure: maximpa exhant epaed, average adiabut exponent, effective exhanat apeed in flight, und eveluation mumber K. From the graph we see that those faels, like Band Li , the boiling point of whose oxide in below the end comperature of conbustion of octane, do nos show the characteristic effect of the dispersion, For them the curves of $C_{\text {max }}, x$, and $G$, shaw no maximun; i.A., the supension is no more favorable than the better of its two components. For the remaining materials, $\mathrm{Be}, \mathrm{Al}$, and Mg , the characteristics at optinum conposition are:
for $B$ - hydrocarbon fuel with $39 \%$ by weight of netal $C_{\text {max }}=4100 \mathrm{~m} /$ nec, $x=1.20 ; C=3400 \mathrm{~m} / \mathrm{sec} ; K=1.51 ; E=1557 \frac{\mathrm{kc} / \mathrm{d}}{\mathrm{dm}}$
for Al - hydrocarbon fuel with $60.5 \%$ by weight of metal $C_{\text {nax }}=3760 \mathrm{~m} /$ sec; $\$=1.189 ; C=3140 \mathrm{~m} / \mathrm{sec} ; ~ W=1.14 ; E=1453 \mathrm{kcel} \frac{\mathrm{km}}{\mathrm{dm}}$
for $\mathrm{Hg}_{\mathrm{g}}=$ hydrocarbon fuel with $80 \%$ by weight dif metal $C_{\text {max }} * 3725 \mathrm{~m} / \mathrm{sec}^{\mathrm{i}}$ $X=1.171 ; C=3080 \mathrm{~m} / \mathrm{sec}^{\prime}: K=1.00 ; E=1330 \mathrm{kc} \mathrm{al}^{1}$
$\mathrm{dm}^{3}$
Surnarizing we can say: Mg dispersed in hydrocarbon has no advantiges over pure hydrecarbon. Al and Be show a wide range of suspensions in which they are superior to the pure hydrod carbon by up to $14 \%$ and $51 \%$ resp. Because they are sasily obtained, Al - auspensions have speciel inportance for military rocket-flight technique, while Be - suspensions come into considaration for special uses. All the stadies were limited to stöchiomotric proportions, to the porsibility still exists that other mixture proportions may reach better valuea of K or C .

The preparation of $60 \%$ Al - gas oil suspensions, which are still usable after many weeks if left untouched, and which are easily fed through centrifugal pumps, was done in two ways on the basis of suggestions by H. Troitasch and E. Husser:

1. Increasing the viscosity of the gasoil by dissolving various materials anch as metallic salts of fatty acids, waxes, fats, rubber or rarious syntheties. Good results were obtained in tests with natural and syathetic rubler, and similar high-polymer hydrocarbons, the oppanols. The oppanols have the further advantage that, being pure hydrocarbons, they require no ballast materials, but burn completely with large heat output.
2. Decrease of particle size of the Al dust while hindering surface oxidation as much as possible, since with decreasing particle size, the sedimentation speed and, in most cases, the viscosity decreases. (?) For these large quantities of metal, pulverizing by using supersonics seemed impractical. So the powder was ground in ball-mills in nitrogen atmosphere.

In connection with the use of liquid $\mathrm{O}_{2}$ as a component of all the rocket fuels discusaed so far, the prohlen of storing very large quantities of this material is important, Because its boiling point is $-183^{\circ} \mathrm{C}$, it will be continuously boiling as a result of the steady flow of heat from its warmer surroundings, and will liberate the energy absorbed by vaporizing with a heat of vaporization of $51 \mathrm{kcal} / \mathrm{kg}$; so that the residual material can maintain itself at this low tenperature. This undesirable vaporization can be decreased by lowering the heat transfer from the surroundings; which occurs mainly through the tank walls. which are wet by the liquid. A first method is the decrease of the wetted surface by putring all the material to be stored into a single tank of spherical shape. The heat flow through this smallest surface can be further decreased by the use of various standard treat-insulation procedures, of which heat-stapping materials like looge powder of magnesium carbonate, with a heat conductivity of $\lambda=0.027 \mathrm{kcal} / \mathrm{mb}^{\circ}$, have shown themselves effective, in the form of thick layers. In the existing temperature range of $+20^{\circ} \mathrm{C}$ to $-183^{\circ} \mathrm{C}$, 131 kesl per square mister of tank surface will go through a 1 meter thick insulating layer during a 24 -hour period; this corresponds to $0_{2}$-vaporization at the rate of 2.57 kg per day per sq. meter of surface. With these figures, the daily loss by vaporization is shown in Fig. 23, for various contoiners up to a million con capacity, for three thicknesses of the insulating layer $1 \mathrm{~m}, 5 \mathrm{~m}$, and 10 m , and taking account of the spatial heat flow through the thick walls.

The results of this calculation were confirmed in a trial installation of a liquid $O_{2}$ tank with 56 ton capacity and a magnesium carbonate insulating layer of average thickness 2.6 m . This tank has been running at the aircraft-testing station at Trau since 1938; the manufacturer is the "Aktiengesellschaft für Industriegasverwertung Berlin - Britz". Although even this tank, (though small as measured by rocket-technical requirements) represents a brand-new development as compared to all previously constructed liquid $O_{2}$ containers, and no experience with tauks of such size existed, the tank worked satisfactorily from the first day it was used. Its vaporization is 140 $\mathrm{kg} / \mathrm{day}$, which is exactly the theoretically expected value, as can be seen by substituting the


Tank contents in tons
Figure 23; Daily evaporation loss from cylindrical large scale containers for liq. $\mathrm{O}_{2}$ for various capacities and various thicknesses of inorganic insulating material.
tank-volume, insulation thickness, daily vaporization in Fig. 23. In the photograph, Fig. 24, can be seen the 8 m . high, Bm. dianeter outside cover of the tank. Inside this steel cover hangs the actual container for the fluid; it is made of brass and has a capacity of $50 \mathrm{~m}^{3}$ or 56 tons. The space between the two metal shells averages 2.6 m in width, and is loosely filled with finely powdered magnesium carbonate. The whole tank stands freely in a subterranean space, where it can be approached and examined from all sides. This room has a completely normal cellar climate; no noticeable drop in temperature can be observed. The outer steel cover of the tank is also at normal temperature; moisture in the air does not condense on it. The 140 kg . of $\mathrm{O}_{2}$ vaporized each day are collected in flasks and are used for operation of the testing station and for welding.

Fig. 25 shows a schematic diagram of a large container for a million tona of liquid, which has daily evaporation of $13,000 \mathrm{~kg}$. for a 10 m . insulation thickness. This centainer, when once filled and left alone, will become empty only after 200 years. The amount evaparated daily can be used by simply filling steel flasks and transferring then to the consumer. The cylindrical inner container of Cu-alloy has a dianeter of 103.5 meters. and, including the arched base, a height of 119 m . The bottom end is conceived of as a hanging floor which rests on a ring running along the boundary of the puter cylinder. This ring is supported on the ground by poorly conducting pillars placed at intervals. The top cover is hung on the outer steel cover at various points. The insulating layer of loose magnesium carbonate powder is around the inner container, and is 10 m . thick on the sides and cover, 15 m . thick below the floor. This insulating layer is bounded and supported by the steel cover which is rigid and strong. The whole thick-walled container stands or a cylindrical subterranean bunker of reinforced concrete.

The liquid $\mathrm{O}_{2}$ comes in through the intake at the left and is led through the intake pipe to the floor of the container, in order to excite as little notion of the oxygen during filling as possible. At the top of the tank is the drain pipe for the vaporized gaseous $\mathrm{O}_{2}$, by means of which normal atmospheric pressure is maintained above the liquid surface. The liguid is removed at the lowest point of the container. At the ana level as this point, a pump system stands on the right at the edge of the bunker, and feeds the liquid to the outlet at the top edge of the bunker.

Further details and auxiliary equipment of the tank will not be mentioned here, but the behavior of the liquid $\mathrm{O}_{2}$ in the large tank requires further consideration. In the liquid pool, which has a depth up to 117 meters, the hydrostatic pressure, as shown in Fig. 25, increases with the depth op to 13.1 atm. The boiling point of $0_{2}$ depends on the pressure, and increases with depth in the liquid from $-183^{\circ} \mathrm{C}$ at 1 atm to $-1440^{\circ} \mathrm{C}$ at 14.1 atm. In spite of thas, the entire contents of the tank will stay at $-183^{\circ} \mathrm{C}$, the temperature of the surface of the liquid. For if the masses of liquid down below should warm to higher temperatures than the upper layers, their density would decrease and a convection would occur which brings the warner masses upward to the region of lower hydrostatic pressures, where as a result of the decreased pressure they will begin to boil, give off the heat of vaporization of the liberated gas, and cool to the temperature of their sarroundings. Since this process holds for layers at any depth, the lowest temperature at the liquid surface will bring all the lower layers to the same temperature. Thus in practice one will actually find the liquid at the bottom of the tank to be at $-183^{\circ} \mathrm{C}$. Since the heating occurs mainly through the side walls, the novement of the fluid will be such that the heated boiling masses of liquid rise along the walls, while the cold masses sink in the middle of the tank so that the toroidal streaming shown in Fig. 25 develops.

Liquid $O_{2}$, whose combustion properties and storage have been discussed in detail here, need not give, in combination with the previously described fuels, the greatest exhaust speed. Therefore, two further notable candidates will be discussed briefly - fluorine and ozone for ozoneenriched liquid $O_{2}$ ).

Fluorine would be used only for metallic fuels. The corresponding metal fluorides in sone cases have higher heats of formation and lower heats of vaporization than the oxides, give the prospect of higher exhaust speeds, but are not considered further because of the technical difficulties in using liquid fluorine in place of liquid $\mathrm{O}_{2}$. The use of pure ozone can be easily checked for the case of cotane. Since ozone has avaíable a disintegration enerpy of $710 \frac{k f a l}{\text { fig }}$, the heat-output of the mixture increases from $2587 \frac{\mathrm{kcal}}{\mathrm{kg}}$ for $0_{2}$ to $3140 \frac{\mathrm{kcal}}{\mathrm{kg}}$ for onone, and the theoretical exhaust speed $c_{\text {th }}=4655 \mathrm{~m} /$ sec increases $3 y 10 \%$ to $5120 \mathrm{~m} / \mathrm{sec}$. The use of pure ozone increases the effective exhaust speed by this same order of magnitude.

To test the applicability of ozone to rockets, H. Schumacher in Frankfurt made tests on a small scale, with the following main results:

Both gaseous and liquid pure ozone are explosive under the working conditions in flipht, and are thus not suitable. Gaseous ozone - $\mathrm{O}_{2}$ mixtures at N.T.P. will react, in pipes or spherical chambers, starting at a $10 \%$ weight fraction of ozone, if che reaction is initiated by an

$\begin{aligned} \text { Figure } 24 ; & \text { External view of liq. } 0 \mathrm{~m} \text { tank of } 56 \text { ton } \\ & \text { capacity, } 2.6 \mathrm{~m} \text { insulation thickness and } \\ & 140 \mathrm{~kg} \text { daily evaporation. }\end{aligned}$
Sand covering
Figure 25；
定 Skatch of an enormous tank for 1 million tons of $11 q . O_{2}$ Brith a diagram of the pressure and boiling point ratio员简in the liquid cantent．
incandescont body, but the reaction will not spread to all of the mass until the ozone concentration reaches $17 \%$ by weight. Even then the pressures developed in the reaction are 3 maderate that they night be expected to require motors and armatures. (?) The pressure increases are of the order of 2 atm. A luminosity starts only at ozone concentrations of $40-50 \%$ by weight, and the pressure increases reach 6-10 atm.

Liquid ozone-oxygen mixtures at atmospheric pressure and the corresponding boiling point seem to have, for weight \% of ozone above $25 \%$, a tendency to explode with great destructive effect, if gaseous ozone above them explodes with emission of light. The miseibility of liquid ozone with liquid $O_{2}$ and the boiling point of the mixture are sufficiently understood from the solubility diagram and vaporization curve at atmospheric pressure, that we assume the following regarding the storage of large quantitits of the mixture in open containers:

Mixtures up to $25 \%$ by weight of ozone are stable, do not separate, and have a temperature of $-183^{\circ} \mathrm{C}$. Since such mixtures seem to be safe from explosion, they are of importance for technical use. Mixtures between 25 and 55 weight \% of oze are not stable; they split into a heavier, deep violet, ozone-enriched phase ( $>55 \% \mathrm{O}_{3}$ ), which sinks to the bottom, and a lighter, light blue, $\mathrm{O}_{2}$-enriched phase $>75 \% \mathrm{O}_{2}$ ), which floats above. The phase which sinks is probably ex plosive. Mixtures with more than $55 \%$ ozone by weight are again stable but in danger of exploding, so they are without imediate technical value.

For zncreased pressures the critical solubility temperature of -179.5 is quickly passed, and a miscibility gap no longer occurs. Liquid ozone-liquid oxygen mixtures which are stored for long periods becone ozone-enriched because of the more rapid boiling off of $O_{2}$, so that the resultant explosive tendency of the tank contents must be counteracted by adding $\mathrm{O}_{2}$. The gas phase above the liquid surface reaches the critical ozone concentration of $40-50$ weight $\%$ only for ozone concentrations $>90 \%$ in the liquid phase, sG that the products of vaporization of liquid ozone $-\mathrm{O}_{2}$ mixtures of up to $25 \%$ ozone content are scarcely dangerous even for rapid vaporization on uncooled machine parts. Fig. 26 shows the behavior of liquid ozone-1iquid $0_{2}$ solutions.

A few other oxidizing agents are mentioned in the first part of this book; two of them, $\mathrm{H}_{2} \mathrm{O}_{2}$ and $\mathrm{HNO}_{3}$ have attained some practical value for certain rockat uses.

Aside fron the three groups of non-self-acting rocket fuels : combustion of hydrocarbons with $\mathrm{O}_{2}$ or ozone, burning of light metals in $\mathrm{O}_{2}$ or fluorine, and the combination of the two groups in the form of light metal-hydrocarbon suspensions, a fourth group of self-acting fuels is possible, which use the heat liberated in formation of molecules from the substance in atomic form: atomic nitrogen (9)
(11)

$$
2 N=N_{2}+6050 \operatorname{tea} / / \mathrm{kg}, c_{\mathrm{th}}=7120 \mathrm{~m} / \mathrm{sec}
$$

and atomic hydrogen

$$
2 \mathrm{H}=\mathrm{H}_{2}+51400^{\mathrm{kro} /} / \mathrm{kg}(11,32) \cdot c_{\mathrm{th}}=20,800 \mathrm{~m} / \mathrm{sec}
$$

Since the life times of these unstable naterials is very short (the life time of active hydrogen is given as at most 10 sec [9, page 253], research in this branch of rocket fuel development must first take the following lines: 1. Finding a basic method for prolonging the life of active nitrogen or hydrogen, 2. Determination of the dependence of lifetime on temperature and pressure, especially in the direction of very low tenperature. 3. If necessary, development of a method for enriching liquid or solid material with the nonatomic modification.

Though the difficulties of such research may be great, and the prospects of technically valuable results are small, it should be noted that, because of the $10-20$ times greater energy concentration of $H$ as compared to presently nvailable fuels, the more favorable specific heat, the higher $\alpha$ values - i.e., greater jet efficiency, the higher reaction velocity - i.e., greater furnace efficiency for smaller furnace volume, and the highly diatheraic behavior = i.e., slight thernal stress on the furnace walls; even partial successes, say a $10 \%$ enrichment of $H$ in $H_{2}$ by perhaps dissolving gaseous H in liquid $\mathrm{H}_{2}$, would be of extraordinary technical importance. For the limiting case of $100 \% \mathrm{~N}$ or H concentration as starting fuel for combustion, the following characteristic quantities were calculated for a furnace pressure of 100 atm :

$$
\begin{array}{ll}
\mathrm{N}_{2}: T_{0}=8260^{\circ} ; \% 1.49 ; \quad \sqrt{3}=0.97 ; \quad c=4690 \mathrm{~m} / \mathrm{sec} \\
\mathrm{H}_{2}: T_{0}=5500^{\circ} ; * 1.49 ; \sqrt{\gamma_{j}}=0.97 ; \quad c=14100 \mathrm{n} / \mathrm{sec}
\end{array}
$$

For atomic hydrogen these values are plotted in Fig. 27 for various flane pressures. This diagram showe clearly the extraordinary properties of active hydrogen; i.e., that the probability of regaining large portions of the dissociation enerky by after-burning in the jet is very larec.

Region of mutually non miscibility
(mixture separates into 2 phases of non-equal density)

Practical, immediately usable nondangarous region.

Regton in which an explosion in the gasectus phase chan to the liould phas so that also the Iiquid phase can explode.

Critical temperatas of solution Ozone boiling poin

Fig. 26: Behavior of maxture of liquid ozone and liquid oxygen at 760 mm pressure according to Schumacher-Frankfurt.


Pigure 27; Dissociation, heat content, cumbustion temperature, theoretical and maximum exhaust velocity and adiabatic exponents for fire gases in the case of association of active H2 to molecular $\mathrm{H}_{2}$ with static cquilibriun.

Finally another group of fuels is worthy of note: These fuela based on nuclear reactions can result in exhaust speeds of $10^{6}-10^{8}$ a/sec! and have recently because of the reactions in uranilum fissien, moved inte the demein of beehnicel intereoto (3).

Sumarizing the mumerical results of this section concerning the problem of exhaust speed we may say that by means of stächiometric combustion of hydrocarbons in $a_{2}$ in rocket motors at 100 atm . furnace pressure, exhaust apeeds aboye $c=3100 \mathrm{~m} / \mathrm{sec}$ are possible. For excekt fuel, 5\% higher values are obtained. By enriching the $O_{2}$ vith oxone, a further increase of exhaust speed to $\mathrm{C}=3400 \mathrm{~m} / \mathrm{sec}$ may be possible vith tochometric mixtures. The use of Al - hydrocarboa suspensions with liquid $O_{2}$ gives similar exhayst speeds, but more favorable proportions by weight on the aircraft, because of the higher fuel density. We ney expect exhaust speeds, of rocket motors in flight, over $3800 \mathrm{~m} /$ sec for liquid $\mathrm{H}_{2}$ with liquid $\mathrm{O}_{2}$, and over $4000 \mathrm{~m} / \mathrm{sec}$ if ozane is included, while the addition of atomic hydrogen would give even higher values, for calculations of flight - and military perfornance of the rocket bomber we thall use the values $C=3000 \mathrm{~m} / \mathrm{sec}$; and $c=4000 \mathrm{~m} / \mathrm{sec}^{-}$To see the effects of higher exhaust speeds we shall calculate with $C=$ $5000 \mathrm{~m} /$ sec for comparison.

## 3. Properties of the Air-Frame

The external appearance of the rocket homber is shown in Figs. 28-31 and discussed theoretically in the next section. The bow of the nircraft's fuselage consists of an "agival" with 9.6 caliber radius of curvature, which is cut by a plane through its long axis so that a flat underside results for the fuselage. Between the wings the semi-agive goes over into a roomy chamber with perpendicular side walls, while the fuselage gradually tepers coward the stermwith a steady decrease in cross-section. The large blunt end surface at the stern of the fuselage is necessitated by the size of the mouth of the jet of the rocket motor. The relatively amoll wing stumps serve manly for stabilization in flight, and for landing; the wing croas-aection is the well-known triangular wedge profile with a maximum thickness of $1 / 20$ of the depth at $2 / 3$ of the wing depth. (18, p. 170). To this peculiar aircraft shape there correspond the laws of flow for very high Mach numbers. An angle of incidence between fuselage and wings is unnecessary, so that for the low-wing arrangement chosen, the lifting flat surfaces of the fuselage and wings go orer into each other without a break, as can be seen most clearly in Fig. 31. For the tail surfaces, a symetrical quadrangular cross-section was chosen, which also has a greatest thickness of $1 / 20$ of its depth in the last third of ite depth. The whole arrangement of the tail aurfaces is independent of the streaming from the rocket. jet, since use of the rocket motor and flight below sound velocity never occur together.

The size of the rocket bonber was chosen as a compromise between a series of contradictory requirements. The idea of making the aircraft as large as possible is suggested by the fact that then the ratio of additional load to weight when empty is generally more favorable, that the construction of larger rocket motors is gimpler, that with increased size of aircraft the military strength of a rocket bomber group increases while the number of capable pilots required per unit of load transported decreases. If one computes a few comparison designs in the range of 10-100 tons starting-weight, one finds that with increasing weight of the aircraft, the aerodynamical lifting power contributed by the fuselage represents (for geowetrical reasons) a smaller part of the total weight, so that the wings have to be relatively larger; finally the weight of the wings predominates, without giving any noticeable improvement in gliding angle in the region of high Mach numbers. Such considerations lead to an apparently favorable takeoff weight of 100 tons, to which corresponds an empty-weight on landing of 10 tons. Thus a limit of 90 tons of fuel with about $76 \mathrm{~m}^{3}$ tank space must be included, which leads to the fugelage dimensions shown in Fig. 28.

The wing dimensions are determined by the permissible wing loading of the bomber. The starting procedure by rocket catapult, which has already been described briefly, pernits practically high wing-loading; thus even though before landing the consumption of all fuel and remoral of all ballast reduces the weight to $1 / 10$ the takeoff weight, the landing speed determines the wing size. Though landing speeds of over $200 \mathrm{~km} / \mathrm{hr}$ can be used in special cases, a permissible limit of $150 \mathrm{~km} / \mathrm{hr}$ was first chosen because the landing of the rocket bomber is to be considered a glide-landing, and because one camot count on the availability of experienced test pilots. The corresponding stagnation pressure is $q=110 \mathrm{~kg} / \mathrm{m}^{2}$. From Fig. 34, the very thin and slightly curved wing profile leads one to expect a maximum lift coefficient of onlv $\mathrm{C}_{\mathrm{a}}$ max $=1.25$ even with landing aids, so that the wing-loading before landing is limited to of $\mathrm{C}_{\mathrm{a}} \max =1.2 \mathrm{man} .5 \mathrm{~kg} / \mathrm{m}^{2}$. Aside from this figure, the wing size in determined by the fact that the fuselage plays an inportant role in the lifting power of the whole aircraft. According to the investigations of the next saction, for large Mach numbers $2 / 3$ of the total weight is carried by the fuselage, and $1 / 3$ by the wings. At landing speed, the lift coefficient of the fuselage, at the angle of attack for maximiun lift, is $C_{a}=0.45$; with the already fixed fuselage supporting surface $F_{t}=80.8 \mathrm{~m}^{2}$ the total lifting power is $\mathrm{F}_{\mathrm{k}} \cdot \mathrm{C}_{\mathrm{g}}, q=4000 \mathrm{~kg}$, while the residual landing weight of 6000 kg goes



Fig. 29: Top view of rocket bomber of 10 ton empty weifgt.

Fig. 30: Outer form of Pocket Bomber of 10 tons empty Weight, perapective viell from front-above.


F1g. 31: Perapective Fiew from "backwards below of 10 ton onpty weight Rocket bomber.
to the wings. Thus the required wing size is about $44 \mathrm{~m}^{2}$; according to Fig. 28, the totial supporting surface is $125.5 \mathrm{~m}^{2}$; the mean wing-loading is $10,000 / 125.5=79.7 \mathrm{~kg} / \mathrm{m}^{2}$ and the lift coafficient for landing is $C_{a}=A / \Phi F=0.74$; in agreement with Fig. 34.

At cake off, the mean wiag-loading is cen times as high, i.e., $797 \mathrm{~kg} / \mathrm{m}^{2}$; for the assumed slimbing speed of $500 \mathrm{~m} / \mathrm{sec}$ the stagnation pressure at takeoff is $15930 \mathrm{~kg} / \mathrm{m}^{2}$ and the total lift coefificient $C_{a}$ is 0.05 , corresponding to an angle of attack $\alpha=3^{\circ}$, while the lift coefficient at optimum gliding angle is $C_{a}=0.173$ for $v /{ }^{2}=1.5$. At a $3^{\circ}$ angle of attack, $38 \%$ of the total, weight of $10^{5} \mathrm{~kg}$ falls on the fuselage, and $6 \%$ on the wings; the wing loading is thus $1390 \mathrm{~kg} / \mathrm{m}$ 4fter the aircraft rises to the optimum angle of altack of $8^{\circ}, \mathrm{C}_{\mathrm{a}}$ becomes 0.173 ; thus the lift for hauling the craft at $V / a=1.5$ is 346000 kg , of which $48 \%$ rests on the wings with $C_{a F}=0.083$, corresponding to a wing loading of $3720 \mathrm{~kg} / \mathrm{m}^{2}$ in this state of flight which determines the wing stress.

Other cages of loading are important for various parts of the rocket bomber; e.g., for the fuel tanks it is the acceleration at takeoff; for fuselage and pilot it is the acceleration at the end of the climb; for fugelage and landing gear it is the landing which is important.

In eatimating the ratio of enpty-woight $G$, attor conaumption of all fuel and dropping of the useful load, to the flight weight $G$, inmediately after rising from the ground, we started from the known weight distribution for overloaded long range aircraft: airframe 18\%, power plart: $13 \%$, auxiliaries $3 \%$, additional load $66 \%, 80 \mathrm{G} / \mathrm{Go}_{0}=0.34$. The main parts of the rocket bomber were, estimated as: cabin - 500 kg , rocket motor - 2500 kg , wings -2500 kg , a tatal of 56 $\mathrm{kg} / \mathrm{m}^{2}$; fuselage 3250 kg , tail, landing gear, bonb-bay, etc.., altogether 1250 kg ; thus the total weight of the aircraft is 7000 kg , whereas flight weights for ordinary bombers of gimilar size are about 3000 kg . The reason that the rocket bomber with its 10 times greater takeoff weitht is only 2.3 times as heavy as ordinary bonbet of the same size is mainly because the support. ing surfaces, especially the wings, carry not 10 tumes but only $3-4$ times the weight, while the remainder is directly supported by the air without any intermediary structure; moreover because of the definite way of starting and climbing the factor of safety for the racket bomber need be only a small fraction of that for an ordinary bomber. Thus the total weight distribution f the rocket bomber is airframe $7 \%$, power plant $2.5 \%$, auxiliaries $0.5 \%$, additional load $90 \%$, 80 $G / G_{0}=0.1$, All these considerations are valid for scaled-up weighta. One thus obtains directly the correct performance figures, if the bomb load is diminished by the excess weight, above 10 tons, of the aircraft.

Figs. 32 and 33 show an overall schematic of the rocket bomber.
The front view of the craft does not show the retractable front wheel, which operates in conjunction with a retractable tail skid and the landing gear which is retractable into the fuselage betweth the wings. The front wheel serves to prevent dangerous contact with the ground of the bow end during the bouncing motion of the aircraft during landing, and to slow down (with the aid of the landing gear) as quickly as possible the aircraft which comes on to the ground at $150 \mathrm{~km} / \mathrm{hr}$ and has practically no wind resistance then because of the amall wings. Behind the bow is the pressure-tight cabin, in which the single pilot sits. It is tight for inside pressures of $0.4-0.5$ atm. with vacuum outside, and should permit rapid exit of the pilot in case of danger (e.g. after takeoff). Because of the smooth external shape, visibility from the cabin is very poor. In free flight at high velocity, side view slits and optical aids are sufficient, for landing a kind of detachable windshield can be used, since than the pressurization of the cabin and maintenance of the bullet-shape are unimportant. A further easential arrangement for the cabin is that the pilot's seat be so arranged that che pilot can take up the high accelerations along the aircraft axis in the best possible position, so that not only body and head, bat also feet and arns have good supporting surfaces, and at the same position can be shifted. The remaining equipment of the pilot's cabin - instruments. $D / F$ and radio equipment, ventilation, etc. is not considered further. At the back of the pressurized cabin are the tank installations, which consist of two large tubes 20.5 m . long and with maximum diameter 1.8 m ; these constitute the main part of the fuselage. The upper fourth of the tubes' circumferance forns the skin of the aircraft, while the lower half and the space between the tubes is covered so that the required shape is obtained. For constructional reasons the tank tubes are subdivided into the actual containers by means of cross-walls. The purpose of the cross-walls is first to have separate contaifers of correct capacity one behind the other for fuel and liquid $O_{2}$ and that each fuel shall lie symmetrically with respect to the axis of the aircraft, so that thermal stresses and tuists are not developed due co asynmetries; second, as a result of the subdivision, to lower the liquid pressure on the rear end of the tank during acceleration and to prevent the aircraft from becaming tail-heavy as the tanks are emptied; finally the cross walls give the thin-walled fuselage the stiffness necessary for taking up the torques at the roots of the winc-spars. It is advisable to put the oxygen tanks in the front end of the fuselage so that the force driving the $\mathrm{O}_{2}$ to the



## Figure 33; Total view of 10 ton Rocker Bomber

pumps will be as large as possible. Between the wings and the tanks is the bomb bay, suitable for projectiles up to 30 tons in weight. When the bombs ate released, the floor of the bomb bay must be opened completely for a short time so that the axis of the bomb is parallel to that of the aircraft, since in the perpendicular position premature decionation may occur. Just before the main spar meets the fuselage there are chambers for the extendable landing wheels; these chambers are partly on the vertical sidewalls of the fuselage and partiy recessed into the tank cylinders.

Finally, Fig. 33 show the rocket motor at the end of the fuselage:

## 4. The Gllde-Number of the Alr-Frame

During its flight the rocket bomber goes through velocity ranges with entirely different flow characteristics, e.g., the ordinary subsonic range, the supersonic range up to three times the velocity of sound, the domain of large Mach numbers $V / a$, in which Newton's law of air resist ance is valid-i.e., the aerodynamic forces vary with the square of the velocity and the angle of attack; and finally the range: of gaswinetic streaming with very long free paths of the air molecules aid thus special laws of air resistance which are still similar to Newton's.

In all these regions the aircraft must have sufficient stability characteristics, whereas the glide-angle is important chiefly in the Newtonian region for both density types, for if the rocket bomber at the beginning of its glide had Mach number 30, then $99 \%$ of its kinetic energy, would be consumed in the Newtoman region, and only $1 \%$ below a Mach number of 3 .

In the region of higher densities the glide angle an be estimated at only a few special points on the Mach scale, say $V /{ }_{f}=0.1 ; 1.5 ; 3$ and $e$, whereas for rarefied air closed gaskinetic formulae can be given.

For a landing spead of $150 \mathrm{~km} / \mathrm{hr}$, i.e., $\mathrm{V} / \mathrm{a}=0.12$, polars of the bomber and the wings were determined by wind tunuel tests on a $1: 20$ scale model; the results are shown in Fig. 34. The best reciprocal glide number is $1 / \xi=7.75$ for angle of attack ${ }^{\circ}=5^{\circ}$; the best lift coefficient is $C_{a \max }=0.575$ foros $=16.5^{\circ}$.

For $V / a$ between 1 and 3 the difference in pressure compared to the pressure of still air is, for two-dimensional flow at small angles of attack,

$$
\Delta p / q= \pm 2 \alpha / \sqrt{r^{2} / a^{2}-1}
$$

according to Ackeret and Busemann; for spatial flow around the rotation-symmetric end of a cone

$$
\Delta p / g=2 \alpha^{2} \ln _{\text {a }}(\text { fatg orc sin } q /) / a /
$$

according to Busemann and $V$, Kármán, and the tangential friction stresses are $\tau \%=0.072\left(\frac{1}{\mathrm{Re}}\right)^{0.2}$ according to V. Kärmán. With the aid of thest relations the values given in Fig. 35 and 35, for polars of the wings, fuselage and complete frame of the rocket bomber at $V / a \approx 1.5$ and 3 , can be computed. From the Figs. we read off the best reciprocal glide number $1 / \mathcal{E}=3.94$ for $V / a=1.5$ and $\alpha * 8^{\circ}$ and ${ }^{1} / \varepsilon=3.83$ for $V / a=3$ and $\alpha \geqslant 7^{\circ}$. Wind tunnel measurements of the polars, which are feasible in this doman, could not be carried out. In the calculation a total vacuum was assumed behind the stubby stern-surfaces. Actually, these surfaces; in this relocity range where the motor is off and they are not in contact with the flame, cari be acted on by oticeable pressures which make the wind resistance appear to be less than when the motor is going; this must also be noted for wind-tunnel measurements.

For $V / \rightarrow \infty$ the thermal velocities of the air molecules and the pressura of the undisturbed air are respectively negligible compared to the flight speed and the dynamic pressure on the surfaces struck by the air stream. The part of this normal pressure $P_{\text {p }}$ which arises from molecules bouncing into the wall can be taken directly from Newton's law for inelastic collisions: $p_{1} / q=2 \sin ^{2} \alpha \quad$ Whether there is further contribution to the pressure depends on how these molecules leave the surface. For dense air this must take place along the surface of the plate. If the wall were flat and the density of the gas layer which streans away were infinite, then the air molecules coming off would receive no acceleration perpendicular to the plate and there would be no further pressure contribution. Actording to V. Karman the ratio of density increase $\Delta \rho$ to density $\rho$ hefore collision is $\Delta \rho / \beta-2 / \Delta x-1 /$ when the correspanding air pressure ratio is $4 p / p=\infty$ so this case can occur only for $x=1$. For $\alpha=1.4, \Delta \rho \rho=5$, the air density at the plate is 6 tines as large as for still air, the layer has a finate thickness, the angle of impact of the air is greater than the angle of aftack, and the air pressure $p$ is greater than $p^{\prime}$; according to Busemani $p / g=(\ddot{K}+1) \sin ^{2} \alpha$. As far as the value of $\alpha$ is concerned, the molecular collisions in the flow of the condensations are sufficient in number to fully excite all molecular rotations; i.e., $\boldsymbol{X}=1,4$. If the high stagnation temperatures necessary for


Figure 34; Wind tunnel measurements at Regnolds' Number $4 \times 10^{6}$ and Mach number corresponding to the landing speed. (The aerodjnamic coefficients are based on the area of the supporting surfaces.)


Figure 35; Calculated polars of wings, fuselage, and of the entire frarie of the-Rocket Bomber in the gas dynamic region and $\mathbf{M}=1.5$. These aerodynamic values are based on the area of the supporting surfaces.


Coefficient of Drag Cw

Figure 36; Computed polars of the wing, the fuselage, and of the ontire frame of the Rocket Bomber at $\mathbf{M}$ 3. (The aerodynamic coefficients are based on the area of the lifting surfaces.)
excitation of molecular vibrations are reached, then considerable oscillation can occur an the available streaming time, ice., © approaches 1.29 . For $\mathcal{X}=1.4$ the pressure on a flat surface is $P / q \doteq 2.4 \sin ^{2} \alpha$. The corresponding pressure in the case of a cone with stream along ins axis is, according to Busman and Guderley, $P / q \doteq 2.15 \mathrm{in}^{2} \alpha$. If the wall surface is curved in the direction of the streaming, the layer streaming off along the wall must follow this curvature and undergoes acceleration perpendicular to the plate, so that for convex curvature a negative additional pressure $\rho_{2}$ results. For $2=1$, these were calculated by Busemain for general curvacures. For the important case of constant radius of curvature the equations can be integrated and give for the lens profile $-\rho_{0} / 9=\sin ^{2} \alpha_{s}-\sin ^{2} \alpha \quad$ and for the ogive-shape, $-A_{2} / q=\frac{B}{3} \sin ^{2} \alpha_{s}+\frac{1}{3}\left(1-\cos \alpha_{5} \cos \alpha\right)-2 / 3 \sin ^{2} \alpha$ where o $x_{s}$ is the angle of attack for the first surface element. Thus the air pressure drops very rapidly as we move back from the end of the object, and vanishes, in the case of the lens profile, at the paint for which the angle of attack is $\alpha(\$ / \sqrt{3}$; for the bullet shape it vanishes somewhat later, at $\mathscr{I} / \sqrt{8}$, so that the average pressure on the curved surfaces is far less than for plane surfaces. For $\mathscr{2} / /$ the centrifugal effects, according to Busman and Guderley, are somewhat larger, so that the pressures for of $=1.4$ are $7 \%$ smaller than ford $=1$, in the case of the lens profile.

As regards the frictional stresses parallel to the wall between the fixed surface and the air, for dense air, momentum parallel to the wall is transferred to the wall only by a thin boundary-layer of molecules near che wall, so that che usual friction laws are valid. One is led, in the case of high supersonic velocities, to give main importance to laminar frictional effects, so that the friction is determined by V. Karmans formula $\sigma / 9=13 / \sqrt{6}$. In the valleys of the flight path of the aircraft, which determine the energy consumption, we may use a value of $\mathrm{Re}=10^{8}$, which gives $\tau /{ }_{q}=0.00013$ for flat, intilted surfaces. At finite angles of attack, the density, friction and temperature change on the leeward side (negative angle of attack) become zero, while for the windward side they are 6 times as great as for free air; at she same time the viscosity of the air increases with the stagnation temperature $V^{2} \sin ^{2} a / 8000$ according to the relation

$$
\left.P=1.753 \times 10^{-6} /\left(1273+1 \xi^{3} \text { in }{ }^{2} / 2000\right) / 275\right)^{a 76}
$$

The frictional forces on the surfaces to windward are approximately

With the aid of these equations we can calculate the aerodynamic forces on an arbitrary body for $V_{0} \rightarrow \infty$. Fig. 37 shows the polers first for the infinitely thin flat plate which is known to be theoretically the best wing for flight above the velocity of sound, second for the wedge profile with flat sides and a thickness $1 / 2$ of the wing depth in the second third of the profile ( $\mathrm{B}, \mathrm{p} .170$ ) and finally for the symmetric double-convex lens profile composed of two equal circular arcs, also with thickness $1 / 20$ of the depth. In the region of $V / a=1$ to 3 , where the linear dependence of the aerodynamic forces on the angle of attack is valid, and where the excess pressures on the windward surfaces and the subnormal pressures on the leeward surfaces are of the some order of magnitude, the biconvex lens profile gives the best glide-numbers; in the Newtonian region, where the air pressure varies quadratically with the angle of attack, and where the air pressure vanishes in the shadow region, the flatesurfaced wedge profile is definitely superior. In the region of angles of attack which are greater than the front bevel angle of the wedge, it is as good as the infinitely thin flat plate. In addition it has the remarkable proparty, in this velocity range, that the ordinarily strict requirement of minimum profile thickness is weakened, in the sense that even the limiting profile can be so thick that the whole wing surface is in the shadow. The inferiority of the lens profile to the wedge profile in this velocity region arises from the fact that larger angles of attack give poorer glide-numbers; since the pressure depends quadratically on the angle of attack, the large angles for the front parts of the surface outweigh the effect of the small angles at the back parts so that altogether a poorer glide-number results than for the flat underside of the wedge profile. A further reason for the inferiority of the lens profile is that the curved surface, because of the centrifugal action, is acted upon by miller normal pressures and about the same frictional stresses as for the wedge profile; thus to obtain the sate lift, larger surfaces must be used, which involves not only greater weights but also greater frictional forces. Since the rocket bomber needs the best possidle glide number at high velocities, it should be designed with wings having a wedge profile.

These considerations can in principle be extended to spatial flow. Fig. 38 shows the molars for 3-dimensional flow around three surfaces with equal projected area and the same height which might be considered for the bow shape - a circular cone with height four times the diameter of the base, a bullet with the same height and base diameter ( 16.25 caliber radius), and finally - half-bullet with the same height and a semicircular base of the same size as that of the two surfaces of revolution, ie, with 8.25 caliber radius. The aerodynamic coefficients are all for


Figure 37; Polars of a plane curface, a wedge profile and a
 suriaces)
the same projected area. While the cone and bullet differ littie in glide-number, the halfbellet with flat surface to the front, is far superior, having a glide-number with $/ E \cdot=4.12$, This is of decisive importanee for the shape of the fuselage of the rocket bomber. White in the region $V / a=1$ to 3 , the bullet has the optimam glide-number, the laws determining the shape of a body under 3 -dimensional flow change for high Mach numbers in the same way as for the wing aections; on the under side of the surface, which mainly determines the magnitude and direction of the net acrodinamic force, the air pressure should be as large, and the resistanct componento as swall, as possible. The underside should consitt solely of surfaces tilted toward the course wind. For a given average angle of attack of the undersurface, and for quadratic dependence of preasure on angle of attack, one obtains the best fatio of drag to lift on the underside if it is curved as little as possible in the direction of flow. On the upper side of the body, which with proper shape is unimportant for determining the total aerodynamic force, the pressures should be as small as possible. Therefore, the apper side should preferably consist only of surfaces tilted to leeward. If this is not possible, the parts of the upper surface which are to rindward should have the smallest possible angle of attack, and be curved convex to the direction of flow, to keep the streaming pressure low by taking advantage of the centrifugal effect of the airmass streaming off the curvad surface. Sumnarizing, for large Mach numbers, bodies should be so streamlined with a point or dikedral at the front, that the lifting undersurface is not curved in the direction of steaming, and that the upper surface should consist as fir as possible of surfaces to lecward; the unayoidable windmard sections of the upper surface should have a convex curvature in the direction of flow. (29)

The fuselage of the rocket bomber was shaped in accordance with this recipe; its polars
 wings and fuselage are $\alpha=4^{\circ} 20^{\prime}$ and $\alpha=8^{\circ} 30^{\prime}$ resp. Since the wing surface is smaller than the fuselage surface, the more favorable glide-number for the wings has little effect compared to that of the fuselage, and one finds a theoretically most favorable angle between wings and fuselage of $-2^{\circ}$ for which the optimum reciprocal glide number $/ E=6.51$ is obtained for angle of attack $\alpha=5^{\prime} 30^{\prime}$ for the wings and $\alpha=77^{\circ} 30^{\circ}$ for the fuselage. For constructional reasons, and also to hove simple streaming conditions at the wing roots, the angle of incidence was chosen as $0^{\circ}$; the polar for this case is show in Fig. 36, which shows a best $/ / E=6.4$ ford $\left(-7^{\circ}\right.$. This value of glideanumber, which can be attained by proper shaping of the fuselage and wings, is surprisingly farorable for high supersonic speeds, and is scarcely different from that of present aircraft below the velocity of sound. We may safely assume, for the velocity range $\frac{y}{b}=30$ to 10 , that the results derived for $\not / 0 \rightarrow \infty$ still apply, and that below this range the characteristics for $\frac{V}{\mathbf{E}}=1$ to 3 gradually appear.

On the basis of the previous investigation the variation of glide number of the rocket bomber for various angles of attack can be represented as in Fig. 40 for the whole range of velocitie: in dense air. The very favorable glide-number below sound-velocity, $\varepsilon=\frac{1}{75}$ at $\alpha x=5^{\circ}$, drops very rapidly as we approach the velocity of sound, then takes on the value $\overline{\mathcal{E}}=\frac{1}{\mathcal{F} \delta}$ at $\alpha=8^{*}$ for velocities slightly above the velocity of sound. Above $\frac{y}{2}=3$, the glide-number improves and then rapidly approaches its favorable behavior for high Mach numbers, with

$$
E=\frac{1}{6.4} \text { of } \alpha=7
$$

In all the previous considerations on the acrodynamic forces, the air was considered as a continuous medium. As the later sections show, the path of the rocket bomber reaches heights of over 1000 km . There the density of the air becomes so very small that on the one hand the stagnation pressures for even extreme velocities no longer result in aerodyriamic forces it all comparable with the other external forces on the aircraft, and also the laws for calculating the aerodynamic forces are no longer those derived for a continuous medium. Fig. 41 is a plot of the air density $\rho$ as a function of the height'H ior values up to $20,000 \mathrm{~m}$ from the well known formula for a homogeneous isofbermal atmosphere; for heights between 11 and 22 thousand $m$, the relation is $\mathcal{D}^{2} / 6=46839 e^{-2 / 5 s d /}$ (see also "Dinerm 5450" or (33)). This expression for density at
 tions. It is frequently assumed thet the composition of the atmosphere at high altitudes is the same as on the ground - i.e., mostly $\mathrm{N}_{2}$ and $\mathrm{H}_{2}$, but that these materials are mare or less dissociated into atoms. Therefore two dotted curves in Fig, 41 are used to show the density if all molecules are dissociated into atoms (left curve), and if only a partial dissociation occurred, as assumed by Godfrey (middle curve). The molecular free path for constant composition varies, with the density according to the relation $2^{7-7 / 2} 10^{-7} / / / 8$. This relation also holds approximately for the docted curves for the dissociated atmosphere, since the effect of dissociation on the free pach is of the same order as the error in determination of the free path. At heights of $40 . \mathrm{km}$, the free path is already greater than the thickness of the laminar boundary layers ( $10^{-4}-10^{-5}$ ), reaches the size of the aircraft dimensions at a height of 120 km ., and rises to over 1000 km . at a height of 200 hm . The stagnation pressure corresponding to the "velocity of escape" $V=12000 \mathrm{~m} / \mathrm{sec}$ is less than $1 \mathrm{~kg} / \mathrm{m}^{2}$ at 100 km . altitude; i, e, at 100 km . altitude the aerodynamic forces have practically disappeared, and the motion of the rocket bomber is alang practically a pure inertial path. The expressions derived for the aerodynamic forces on the assumption of a continuous medium are already iovalid at heights aver 40 km , since the distances


Fig. 38: Polars for three bodies having equal height and ecual base crosssectional area.

1. A right circular cone
2. An ogival shape
3. A half ogival shape
herodynamic coefficients are based on the base area. Mach number infinite. Gas dynanic regior.


Figure 39; Computed polang of the wings, the fuselage, and the entire Prame of the Rocket Bomber at ME3 (The aerodynamic coefficients are based on the area of the lifting surfaces).


Figure 40; Probable behavior of lift-to-drag ratio and best angle of attack of Rocket Bomber ait $0.1 \leq M \leq 30$.

Log of the free path length in meters


Fig. 41: Variation of the specific weight of the atmosphere, and the Iree path length with the flight height H.
between air molecules are already larger than the dimensions of the streams assumed in the calculation, e.g., the thickness of the boundary layer. For such rarefied air, no procedure for calculating aerodynamic forces is known. Only when, ror oltitudes above I00 kn, the Free pacna approach the dimensions of the moving body, do the relations simplify sufficiently to enable a fairly exact estimate of the aerodymanic forces. (28)

In this region of gaskinetic streaming the following equations are valid for the aerodymaic forces on a body moving at arbitrary veloc_ty $V$ at altitudes above 100 ko , where the upper (lower) sign refers to negative (positive) anges of attack:

 before collision with the wall, $\mathrm{C}_{\mathrm{R}}=\sqrt{2 g} \mathrm{~g}^{2}$, i the most probable diffuse recoil velocity of the

 $A^{\prime}$ presentation of these aerodynamic forces $2 n$ closind form as a function of Uach number is not
 ing relation between $\mathbf{C}_{\boldsymbol{p}}$ and a .

All the equations refer to diatonic gases or gas mixtures. Also they require the knowledge of the characteristic temperature 6, and thus involve nore special assumptions than do the gas-dynamic equations which require only a knowledge of the no. of atoms per molecule and the sound-velocity, since they invelve only $O$ and a.

The equations for the momentin ipf of the molecules colliding with the wall, and for the tangential momeritum if given to the wall on recoil are taken from (28). The recoil momentum murt be considered in more detail.

Before an expression for the recoil mmentum can be set up, one mast adve a clear picture of the recoil of the molecules, which have a randon thermal velocity $c$ and a relative velocity $V$ making nn angle $O$ with the plate. One must decide whether a bundle of molecules, striking the plate with a perpendicular velocity component $V \sin \alpha+C_{x} \cos \rho \theta$ leave the plate by specular reflection or under some other definite or randon angles; further, whether the collision velocities are conserved for each individual molecuie or have a definite relation to the recoil velocities, or whether they are completely random so that we can derive relations only for the average or most probable speeds, on the basis of energy considerations. One should investigate whether, during the collision, she velocities $\mathcal{C}_{x}$ as well as $V$ are conserved, or whether they interchange their kinetic energy in whole or in part, or whether the internal degrees of froedon of the air molecules or wall molecules take part in this exchange. These questions were answered on the basis of a paper of Frenkel on "Theory of Adsorption and Related Phenomena" (4), whose opinion we give verbatim: "No matter how small the time of adherence, gas molecules colliding with a wall do not simply bounce off, but are emitted again with velocities which need nat have any definite relation to the original velocity, either in magnitude or direction. The usual pifture of elastic reflections of molecules, from which the kinetic theory of gases starts in deriving the pressure on the walls of the container, is false in principle. If one arrives at correct results in this case, it is because the solid body and the gas are considered to be at the same temperature and at rest relative to each other. Under these conditions, the velocity distribution of the emitted molecules is the same as if they were actually roflected from the wall surface. Actually a quite different picture would result if une bombarded the surface in a definite direc. tion with a molecular beam." Sumarizing these considerations we find that, in setting up a formula for the recoil momentum, we must considef gas whose probable recoil velocity $C_{R}$ is defiviable by purely energetic considerations from the probable velocity Cy before collision, from the systematic velocity $V$ and the wall temperature $T W$. The mass of gas striking the wall per second is:


In the normal gas-kinetic case ( $V=0$ and $C_{f}=C_{f}$ ) the process of emission of the molecules can
 second of $\overline{0}=\frac{\rho}{3} \vec{j}-(H$. For the moving plate $(\forall \neq P), K$ times as many molecules collide each second as for the plate at rest, and just as uany must come off per second if none sticks permanently to the wall. Since according to the resentch cited, the process of recoil is completely independent of the process of collision, and is of the same $t$ ype as for recoil at $V=0$, the only possible difference can consist in a change of the factor $K$ for the colliding molecules $i_{p} R=\frac{\rho . C_{H}}{} K_{C H}$ os hag as $C_{N}=C_{H}$. For the same reasons, the difference between recoil and collision tefiperatures can produce no change in the tructure of the formula for the recoil momen-
 considered further. According to the theorem of Conservation of Energy, for the assumed closed system the sun of the energy $\mathrm{E}_{\mathrm{R}}$ taken away by the recoiling nolecules plus the energy $\mathrm{E}_{\mathrm{W}}$ remaining at the wall must equal the energy $E_{A}$ of the impinging molecules: $\bar{E}_{A}=\epsilon_{w}+\mathcal{K}_{e}$
where the valucs frotand $C$ asc have oniy computiotional significance and are therefore not used in what follows.

Usually the ratio of the energy loss of the impinging molecules to the excess of their initial energy over the temperature level of the wall after collision is called the accamodation coefficient $\alpha$; e.g. the average accommodation coefficient for the whole process has the value $\bar{\alpha}=\left(E_{q}-E_{R}\right) /\left(\xi_{M}-E_{w}\right)$. The accompodation coefficients for the individual degrees of freedom can be written as:

The average accomodation coefficient $\bar{x}$ is built up from the coefficients for the separate degrees of freedom as follows:

$$
\bar{\alpha}=\alpha_{\operatorname{trans}}\left(E_{i A} t_{\text {tians }}-E_{w} t_{\text {rans }}\right) /\left(E_{p}-E_{w}\right)+\alpha \text { rot }\left(E_{\text {prat }}-E_{w r o t}\right)\left(E_{p}-E_{w}\right)+\alpha_{o s c}\left(E_{\text {iose }}-E_{\text {wase }}\right) /\left(E_{p}-E_{w}\right)
$$

Considerations on accommodation coefficients in the physical literature state in essence that the external degrees of freedom take part immediately and completely in the energy exchange during molecular processes, whereas the exchange of internal molecular energy, especially that of vihration, is slowed down considerably and is effective only after long relaxation times, so that the accommodation coefficients for translation are 1 and also $\alpha_{\text {rot }}$ may be assumed to be ©/, while the deviations of the total accommation coefficient $\alpha$ from 1 , observed in practice, are to be ascribed to che slowness of changes in vibration. From $\alpha$ trans $=f$ it follows that $C_{\text {etrans }}=C_{\omega t}$ tons $=\sqrt{2 g R T_{w}}$ where $C_{A}$ trons is thus the desired most probable recoil velocity of the molecules leaving the wall diffusely; $T_{h}$ is the wall temperature in ${ }^{\circ} \mathrm{K}$,

Before calculating the aerodynamic forces one must estimate the wall temperature Tw. This is done by equating the energy radiated by the wall per $\mathrm{cm}^{2}$ per sec. $F_{s}=\xi_{a} \frac{4,96}{3600}\left(\frac{T_{4}}{100}\right)$ to the energy $E_{W}$ transferred by the air molecules to the same surface in the same tinfed If one introduces the previously established assumptions $\alpha_{\text {trans }}=1, \alpha_{\text {rot }}=1, \alpha$ osc $=0$, one obtains $E_{W}=A \bar{\beta}\left(\frac{\gamma_{2}}{2}+\frac{5}{2} Q^{\prime}\left(T_{G}-T_{W}\right)\right]$ where $\bar{D}$ is the mass of the molecules striking unit area of the plate per unit time. These expressions were used in constructing Fig. 42, where the follawing numerical values were assumed; temperature of still air $\mathrm{T}_{\mathrm{G}}=320{ }^{\circ} \mathrm{K}$, density of still air
 tion and dissociation were not considered; also the atmosphere was assumed to be at rest on the earth's surface. The optical absorption of the wall was assumed to be $\boldsymbol{\varepsilon}_{\mathbf{2}}=0.80$. Radiation of heat to the wall from the outside (sun), from the air space or the earth's surface was neglected, os well as from the interior of the aircraft to the wall. Fig. 42 shows that the equilibrium temperature of the plate at, rest is $136^{\circ} \mathrm{K}$ and that this equilibrium temperature of the surface of the aircraft increases with increasing velocity and angle of attack, as long as the latter is positive so that the surface is tilted toward the course wind. For sarfaces to leeward the equilibrium temperature drops to absolute zero, since the heat supply vanishes with the number of impinging molecules. Altogether the cemperatures remann within reasonable linits for all angles of attack and velocities occurring in practice. At greater heights even lower temperatures may be expected; at lower heights a transition occurs to temperatures calcilable by kinetic. theory.

Flight velocity $\bar{\nabla}$ in $\mathrm{z} / \mathrm{s}$


F1g. 42: Surface temperaturg of in flat piate in an atmosphere of $86 \%{ }_{2}$ and $14 \% \mathrm{O}_{2}$ and with density $\rho^{=}=10^{-7} \mathrm{~kg} \sec c^{2} / \mathrm{m}^{4}$ (in molecular flom region) for various flight velocitiss $\mathrm{v} / \mathrm{a}$ and various angles of attack $\alpha$


Fig. 43: Coefficients iph/q of a shock impulse perpendicular to a flat plate for various flight velocities $v$ and angles of attack $\alpha$ in an atmosphare of $86 \%^{\circ} \mathrm{N}_{2}$ and $148 \mathrm{O}_{2}$ and density $\mathscr{J}^{\circ}=10^{-7} \mathrm{~kg} \mathrm{sec} / \mathrm{m}^{4}$ (molecular thor region).

Flight velocity $v i n \mathrm{~m} / \mathrm{s}$


Fig. 44: Coefficient $1 p R / q$ of restitution (rebounding impulse) on a flat plate for various flight velocities $v$ and angles of attack $\alpha$ in an atmosphere of $86 \% \mathrm{~N}_{2}$ and $14 \% \mathrm{O}_{2}$ and density $\rho=10^{-7}$ (molecular flom region).

Flight velocity $V$ in $m / s$


Fig. 45: Coefficient $T / q$ of the thrust tension between air and plane plates for various flight velocities $v$ and varicus angles of attack of in an atmosphere of $86 \% \mathrm{~N}_{2}$ and $14 \% \mathrm{O}_{2}$ and density $\mathscr{J}^{1}=10^{-7} \mathrm{~kg} \mathrm{sec} / \mathrm{m}^{4}$ (molecular flow region).

With the anu oi the equations the aerodynamic forces were calculated for an assuned atmosphere with temperature $320^{\circ} \mathrm{K}$, density $10^{\circ} /$ tgsec $/ \mathrm{m}^{*}$ and conposition $14 \% \mathrm{O}_{2}+86 \% \mathrm{~N}_{2}$. The momentum $z_{p} H$ of the air impinging on the plate is plotted in Fig. 43 for various angles of attack and welocities of flight; the corresponding recoil momentum $i p h$ is shown in Fig. 44, and the tangential momentum in Fig. 45. Since $\dot{c} /=7$, Fig. 45 also represents the coefficient of friction $Z / q$ between the air and the wall, for all angles of attack and for the same range of flight velocities. In this figure, the extraordinarily large coefficient of friction in the domain of gas-kinetic streaming is noteworthy. While values of $\mathbb{T} / 9=0.001$ to 0.003 are customary in aerodynamics, we have in this case values of the friction 300 to 1000 timen an large, and these are not accompanied by corresponding increases in the values of the perpendicular components of the aerodynamic forces. The reason for this astonishingly high friction is the fact that in the domain of gas-kinetic streaming the protectave boundary layer at the surface of the moving body no longer exists. so that all the nolecules which have the chance to transfer perpendicular momentum to the wall, at the same time transfer their tangential momentum completely on the average, whereas this latter momentum transfer is limited to the few molecules inside the boundary layer, in the case of aerodynamic streaming. This very unfavorable behavior of the aerodynamic forces in the rare upper atmosphere would completely prevent flight at these heights, if the stagnation pressures and consequently the forces did not decrease rapidly and finally wanish. Obviously the shaping of the aircraft to fit the streaming conditions loses some of its significance of these heights. Nevertheless, the Newtonian character of the air pressures (i.e. their quadratic dependence on flight velocity and angle of attack), and the rapid disappearance of forces on leeward surfaces, is very apparent at higher velocicies of flight. Thus in the gaskinatic domain all the requirements are fulfilled for the use of streamined bodies, whose underside is curved as little as possible in the direction of streaming, and whose upper side consists, as far as possible, of leeward or convex surfaces, while the thickness of the body is of no importance. Reciprocal streamline-numbers and polars were derived for the flat plate, from Figs. 43-45, and plotted in Figs. 46 and 47. These graphs of the behavior of the aerodynamic forces on a flat, infinitely thin plate, have direct practical importance, because any wing profile of finite thickness, with a flat under-side and upper-side to leeward, undergoes exactly these same aerodynamic forces at high flight velocities, regardless of the shape of the upper side and the thickness of the profile.

Figs. 48 and 49, which show the aerodynamic forces on the rocket bomber in the range of gas-kinetic streaming, were also derived from Figs. 43-45. In the same way as for the gas-dynamic calculations, the actually curved surface of the aircraft was split up into a number of small flat surfaces, and the air forces calculated for each of these component surfaces.

Finally Fig. 50 shows a plot of the dependence of the air forces on height and velocity of flight for the dcmains of gas-dynamic and gas-kinetic streaming. This plot of $p / \mathrm{g}$ against heaght. for the flat plate, for various Mach numbers and with the fixed angle of attack of $7^{\circ}$ (which is the optimum for the gas-kinetic case), shows that the air-force-coefficients decrease with height in this region. This phenomenon is caused by the decrease of temperature of the plate with height, so that ceteris paribus the recoil velocity and recoil momentum also decrease. The forces in the range of heights from $40-90 \mathrm{~km}$ (which cannot be calculated exactly) were interpolated approximately in the form of the dotted curve.

Fig. 51 shows the final result of this whole section, i.e. the dependence of the rocket bomber's glide number (or its reciprocal) on the velocity and altitude of flight, With this we close the investigations on ghide-number of the rocket bomber.

Flight velocity $v i n \mathrm{~m} / \mathrm{s}$


Fig. 46: Ifft to drag ratio of a plat infinitely thin plate in molecular slow ragion.


Fig. 47: Polars of a flat, infinitely thin plate in molecular flow region for various flight speeds.


Fig. 48: Lift to drag ratio of the Rocket Bomber in molecular flow region.


Fig. 49
Polars of the Rociket Bomber in the molecular flow region for various flight velocities.


Fig. 50; Behavior of the coafficient $\mathrm{p} / \mathrm{g}$ of the air pressure perpendicular to a flat plate when angle of attack $\alpha=70$ as a function of filight velocity and flight altitude (conventional fiuid mechanics, molecular flow, and transition region).


Fig. 51;
Rehavior of the best lift to drag ratio of the Rocket Bomber plotted against the flight altitude and flight velocity (conventional fluid mechanics, molecular flow, and transition region).

## II. LAUNCHING AND CLIMB

## 1. Acceleration of the Aircraft

The launching and clinb of the rocket bomber have the purpose of giving it, with a minimum of fuel consumption, the high velocity necessary to carry is through its long glide path; they are in the nature of an impulae which lasts only for a few minutes. Despite this, there are karious possibilities for the variation of this external force during this short time; we wish to find the most favorable, 1, e., the one with the least possible fuel consumption for a given final velocicy.

In Fig. 52, the effective exhaust speed $c$ is again taken as a measure of the useful energy content per unit mass of fuel and the attainable velocity increase $V$ is measured in units of $c_{i}$ the fuel consumption $G_{q}-G$ is measured in units of the starting weight $G_{4}$ ating body the energy available from the conbustion gases $(G, G) c^{2} / \mathcal{G}$, which is theoretically possible in firing from a tube, then $\mathbb{K}=\sqrt{6} / 6-1$ This relation for ideal fire is shown as curve 1 in Fig. 52. In the case of rockret drive, the occeleration process, if it was opposed only by the inertial forees of the bomber and not by air resistance, gravity, etc., would after integration of the equations expressing the. Conservation of the Center of Mass (commomolvo ), be represented by the fundamental rocket equation $1 / 2=4,6 / G$. Curve 2 for ideal rocket acceleration thus gives much smaller final speeds for the same fuel consumption. The energy lost to the acceleration process is that of the motion of the combustion gases relative to the place of launching; this is often called the external efficiency of the rocket drive. This fundamental curve for ideal rocket drive gives no indication of the actugl accelerations. Actually the acceleration of the bomber occurs against air resistance and at least a component of the weight and these resistances become more important for the smaller accelerations. For example, if one assumes that the air resistance and the component of weight along the path together represent a fixed fraction, say $1 / 5$, of the instaneous weight and that the acceleration is constant during the climb, then one can denote by $k$ the ratio of the rocket thrust reduced by the resisting forces to the effective thrust; from the equation $k c d m+m d r=0$, one then derives a modified to rocket equation $\mathscr{C}=\mathscr{K} \ln G / G$ If one takes the rocket thrust to be equal to the instaneous weight, then $4=\left(\mathbb{K}-0.2 / / J=a 8\right.$ and the curve for this type of drive has the equation $S / G=e^{-1 / a b e}$
 not obtain this curve if the aircraft starts with customary small speed at the ground, because for the assumed accelerations the aerodynamic forces would increase mach too quiclly for subsomic velocities. Curve 4 shows the climbing process for a commercial rocket plane with high requirements of safery and convenience, i.e., small takeoff and landing speeds, and small, practically constant accelerations along the path of climb; these are taken from a previous detailed calcalation of the path. (18) The long, tedious, uneconomical climb until che velocity of sound is reached is shown clasily; we see how costly low welocities. at the ground and the sccompanying small accelerations are for rocket flight. Curve 5 shows a possible acceleration process under the following assumptions: the rocket thrust is a times the initial weight $G_{0}$ and stays constant over the whole path of climb; thus the acceleration increases with the decreasing mase of the rocket up to a value determined by the weight of the crev and the aircraft, and the rocket motor is used fully. Thus the weight loss per second is constant and equal to a $G_{0} \cdot g / c$ so that the weight at time $t i s: ~ G=\mathcal{G}_{\varphi}\left(/-\sigma g t_{c}\right)$. By equating the inertial and friction forces (the latter with the same assumptions as for curve 3) to the rocket thrust, it follows that
 $0.25 ; 1.0$ and 10 . In the curve for $P / G_{0}=0.25$ the acceleration increases from $0.5 \mathrm{~m} / \mathrm{sec}^{2}$. to $25 \mathrm{~m} / \mathrm{sec}^{2}$. Because of the small acceleration the curve deviates noticeably from that for ideal rocket drive. For the case $\mathrm{P} / \mathrm{G}_{0}=1.0$, in which thrust and initial weight are equal, the accelerations vary between $10 \mathrm{~m} / \mathrm{sec}^{2}$ and $100 \mathrm{~m} / \mathrm{sec}^{2}$, and the curve approaches most closely that for ideal racket drive. Finally for $P / G_{0}=10$, the thrust is ten times the initial weight, the accelerations lie between $100 \mathrm{~m} / \mathrm{sec}^{2}$ and $1000 \mathrm{~m} / \mathrm{sec}^{2}$, which is certainly way beyond hupanly bearable limits. Thus the very great effect of the acceleration process on the final velocity attainable with a $\bar{f}$ iven $G / G_{0}$ is clear,

The curves for $P / G_{0}=1$ and 10 are based on such reasonable assumptions that noticeably more favorable types of launching could hardly be found; thus more decailed study of the prooesses of takeoff and climb under these assumptions is advissable. The tiwo curves differ only in the accelerations during the climb. The usable accelerations are limited only by the strength of the aircraft and by human resistance. From our present knowledge of the physiological behavior of the man under high accelerations one mast admit that an unmaned rocket aircraft can be driven nt somewhat higher accelerations than a manned craft; nevertheless, the umanned craft also soon reaches the point where greater accelerations are compensated for by the greater construction waight necessary for craft capable of undergoing latge accelerations. In addition the rocket


Fig. 52; Reference curves for the Rocket take-off.
(1) Ideal trajectory "shot"
(2) Ideal rocket take-off
(3) Rocket tako-off with nodarate accoleration against moderate path resistance
(4) Rocket take-off of a slowly starting cominarcial rocket plane
(5) Rocket take-off of a rapid starting rocket bomber with various ratios of thrust to initial weight


Fig. 53;
Limits of accelerations withstood by trained pilots in terms of "G's" as a function of time of curation
bomber, because of its long range and the accurate navigation necessary for the bomb release. cannot do without a pilot on board, Thus the mormissible accelerations are limited to values that can be withstoad by trixined flyera.

In Fig. 53 are shown the resuits of most recent studies ( $2,12,15,31$ ) on the highest aecelesations that are bearable by a man in different body positions, as a function of the length of the acceleration time. Whersas in the sitting position the limit is detennined by disturbanc of the circolatory system especially caused by loss of blood from the brain or heart due to differences in hydrostatic level inside the circulatory aystem, this danger decreasen in the lying position, and the limit then seems to be set by difficulties in breathing as a result of greatly increased weight of the thorax. This more favorable lying position is directly achieved in the seating arrangenent sketched in Fig. 33, since the accelerations due to the engine are taken $u_{p}$ by the pilot in directions perpendicular to the axis of his body. In the lying position in caroussel tests accelerations of 20 g for nure than a minute have been withstood by drugged apes, and accelerations of 17 g for more than 180 sac . by men.

During the takeoff and climb of a rocket bomber there are two phases during which the acceleratıons can reach critical values: the catapplt takeoif and the end of the climbing path. Ihe catapult process at takeoff lasts several seconds; the accelerations can be chosen freely, the acceleration staris rather suddenly, keeps its top value for about 11 sec . and then stops again suddenly, Adjustment of the circulatory system is thus scarcely possible. Because of this impulse and rather long action, one camot go beyond an acceleration at takeoff of more than 5 . even for the favorable perpendicular position. Also higher takeoff accelerations, though they result in economizing on starting-rockets by decreasing the ran on the ground, are unfavorable as regards flight performance, since they necessitate stronger construction of the tank assembly. Conditions are quite different at the end of the climb. During the climb the accelerations increase in the ratio $G$ / $G$ as the result of decreasing mass of the aircraft for canstant motor thrust. Even if the aircraft flies without payload, this ratio will end up. at 10 . This climbing process lasts several minutes, with the greatest acceleration occuring in the last second. The body has plenty of time to adjust itself to the high accelerations, and the highest accelerations last for only a short time. The conditions are thus very favorable, and the effect of the high acceleration on flight performance is also favoreble. Ore can, therefore, go closer to the linita of resistance and, in view of the perpendicular position of the pilot, permit accelerations of 10 g.

For the assumed $G_{(G)}=1$, this means that the factor a equals 1 , i.e., the motor thrust equals the initial weight. The limiting acceleration of 10 g is of course not reathed if the bomber varries cargo. E.g., if it has only $j$ tons of bombs on board, then $G / G_{0}=0.15$ and. the limitint acceleration is 6.67 g . For $P=G_{0}$, the scteleration at the start of the climb is
 if we use the relation $b=g /\left(1-g t_{i}^{\prime}\right)$ which neglects frictional forces. The last increase fron 6.67 g to 10 g thus occurs in the very short time of 20 seconds. Only during these last 20 sec . is the pilot subjected to the critical accelerations, after he has becoore accustomed to the high acceleration gradually and uniforwly during the previous 340 sec .; furthermore, this holds only for the practically unimportant case where he takes off without bombs. With this the assumptions concerning permissible accelerations during climb seem justified.

## 2. Catapult Takeoff

The roket bonber is to be brought to a takeoff speed of $500 \mathrm{~m} / \mathrm{sec}$ by means of equipthent on the eround, in order to increase as much as possible the final velocity attainable with the fuel on board, to minimize the difficulties associaced with the initial wing loading of $800 \mathrm{~kg} / \mathrm{m}^{2}$, and to eliminate the uncertain flight characteristics of the airframe in passing the velocity of sound. For this purpose a catapult-like, perfectly straight, horizontal starting-track several kifometers in length is needed; on this sits a sled, which carries the aircraft, and which is driven by rocket apparatus having a large thrust but moderate exhaust speed. Fig. 54 shows a schematic of the sling arrangement. The downard forces acting along the takeoff path are the weight of the bomber with sied and starting rockets, about 150 tons; in addition there are large forces acting verticnlly upwards or in the direction of the path due to driving and retarding actions respectively. These forces and the necessity for very precise installation of the sliderails suggest the construction of the radbed out of rejnforced concrete, whose cross-section fipr the single-track arrangenent chosen has the isosceles triangle shape shown in Fig. 54; the base of the triangle serves to fix the structure in the ground, while the sled carrying the aircraft ridea on the apex of the triangle. The upper construction of the rail arrangement consists of the main rail lying or top of the wall, which has to take up the perpendicular forces and also transmits the very large horizontal retarding forces, and of two lead-xails halfway up the wall, which support only occasional stnall forces, and which prevent rotations of the whole system of aircraft plus sled around its longitudinal axis. Lubricants of the proper viscosity are used on the main-


Fig. 54; Starting sled of the Rocket Bombers
and lead rails, so that the sliding faces of the sled can glide on them with little friction. The sled itself is mnunted on the sliding faces at $A$ and $B$ and also has a pair of firtings to prevent overturning, The aircraft to be catapulted sits up front on the main body of the sled, the sled's power plant is at the back, behind the aircraft. The high takeoff thrusts are transmitted to the aircraft through the main body of the sled. Very high thrusts of 6.10 tons for 11 seconds and the utmost safety in opeation are required of the starting rockets, while their exhaust speed can be moderate since they do not afkect the flight performance of the bomber. In the use of 40 tons of fuel for starting, one vill not, for reasons of safety and cost, use the selfacting fuels like powder or $\mathrm{H}_{2} \mathrm{O}_{2}$, which are customary for moderate exhaust speeds, but rather the fuels tonsisting of two materials, say !iquid $O_{2}$ and gasoil-water emulsion, are pref. erable. For the talculation of the catapult process, we shall take, as exhaust speed of the ground-based starting rockets, $C=1500 \mathrm{~m} / \mathrm{sec}$, so that the fuel consumption per second is $\mathrm{Pg} / \mathrm{c}=$ $4000 \mathrm{~kg} / \mathrm{sec}$. The weight of the whole apparatus, for constant starting thrust $P$ is $\sigma_{s}=\sigma_{a s}-P t / 6$ at any instant. The air resistance of the whole catapult system is estimated to be $75,000 \mathrm{~kg}$ for $V=1.5$, and increases auring the takeoff according to the equation $W=03$. $V 2$. The sliding iriction of the sled on the takeoff rail is completely negligible compared to the two forses mentioned. Using the second law of motion we can write for the accelaration at any moment during takeoff: $\quad d / / t=G\left(p-0.3 v^{2}\right) /\left(\epsilon_{0 j}-P_{s} t / \epsilon\right)$
If we require that the acceleration at the end of the takeoff shall not exceed $50 \mathrm{~m} / \mathrm{sec}{ }^{2}$, then We obtain for the constant starting thrusta $P$ value of $600,000 \mathrm{~kg}$ if the final weight of the take-off mass is $105,000 \mathrm{~kg}$. By integrating unce, we obtain a relation between time elapsed and velocity atiained $t=G_{05} / 3990 .\left[\mathrm{I}-\left(\frac{429-0.5 \mathrm{~V}}{428+0.5 v}\right)^{248}\right]$

For $V=500 \mathrm{~m} / \mathrm{sec}$ we obtain for the time of takeoffthe law of consumption of the takeoff rocket vields a second relation: $G_{s}=105000=G_{s}-A_{g} H_{L} / L$
from which $t_{1}=10.96 \mathrm{sec}$ and $G_{o s}=148700 \mathrm{~kg}$. The total path length can be obtained most simpl, by numerical integration. Sumarizing, these relations concerning the takeoff process give the results that: the takeoff rockets develop a thrist. P-610 tons for 11 sec, use up 43.7 tons of fucl with $c=1500 \mathrm{~m} / \mathrm{sec}$ if the syaten to be catiopulted weighs 105 tons, the length of the cow path is 2750 m ; and that the acceleratians during towing increase from an initial value of $40.2 \mathrm{~m} / \mathrm{sec}^{2}$ to the permissible limit of $50 \mathrm{~m} / \mathrm{sec}^{2}$ at the end of the takeoff. thus for approximate estimates one way use an average acceleration $b=145 \mathrm{~m} / \mathrm{sec}^{2}$ and colcuiate all the nther quantities; e.g. $t=5 / 6=\frac{500}{45} \pm 1 / \mathrm{sec}$ and $s=\frac{1}{2} 6 t^{2} \pm 2750 \mathrm{~m}$.

Fig. 55 shows the dependence on length of path of the velocity $V$ attained and the residual weight $G_{i}$ also given are some of the data concerning the siding jaws of the takeoff-sled. The two sliding jaws at $A$ and $B$ can be assumed to be of bronze or hard steel and have a square jaw-surface of $0.25 \times 0.25 \mathrm{~m}$; they are a sort of self-adjusting bearing-plate in Michell bearings, and are movable about a hinged edge; the flat undersurface of the jaw its bent up at the front end to assure the flow of lubricant to the edge. From Gumbel - Everling ( $7, P$. 150/151) one obtaink, for a square sliding jaw of side length $t$, if the point of application of the force corresponds to the safest thickness of lubricating layer $\left(x_{1}=0.8\right)$, some fundamental equations; e.g., the coefficient of friction is $\mu=2.37 \sqrt{O}{ }^{2} / p t$. where $p$ is the average pressure $P / t^{2}$ on the jaw; the minimum thickness of lubricating layer is $h_{0}=0.30 t \sqrt{7} \mathrm{~V} / \mathrm{pt}$, and the angle of tilt is $\alpha=0.37 / \sqrt{3} / \mathrm{st}$.

From these, $\mu=7.976$ i.e., the minimum gap $h_{0}$ between the sliding jaw and the rail surface, which make an angle $\alpha$ with each other, should be kept as small as possible to permit large loads $P$ to slide with little frictional resistance. In the desired state of pure liquid friction, $h_{0}$ is limited by the roughness of the sliding surfaces; for the long sliding surfaces of the takeoff track which is subject to rough handline and all kinds of weather, we may expect val. ues of $10^{-4}-10^{-3}$. Thus the minimum gap-height $h$ to assure dynamical buoyancy of the jaws is $h_{0}=10^{-4} \mathrm{~m}$, with $\mu=0,00316$. The largest section-loading ever experienced by the jaws is $p^{0}=75000 / 625=120 \mathrm{~kg} / \mathrm{cm}^{2}$; during takeoff this drops to $4 \mathrm{~kg} / \mathrm{cm}^{2}$. From the previous equations
 7 of the lubricant, decreases in the same manner as $V / \mathrm{p}$ increases; i.e., if 7 Y/p is constant. At the start of the motion of the sled, the sliding speeds are so small that there is no 8 sufficiently large to keep the product constant. In this case, the flat surface of the jaw will lie flat on the surface of the rail and will rise to the angle $\alpha$ only as the velocity increases. Fig. 55 shows the residual part $G-A$ of the weight $G$ which rests on the jaws at any moment; it is assumed that the buoyance $A$ varies quadratically with $V$ from $A=0$ for $V=0$ to $A=100$ tons for $V=500 \mathrm{~m} / \mathrm{sec}$. Thus $P=\frac{C i A}{2}$ and the required lubricant viscosity at any position on the track
 ness $h_{0}$. The rise in temperature $4 T$ of the oil film can be estimated by assuming that all the work against friction goes to heat the oil. Because of the very brief duration of this heating process, the heat conduction from the oil will be extrenely snall. From $A \rho_{\mu} \mu v=t h_{4} v / c_{s} \Delta 7$ it follows that the heating is independent of the velocity; taking values of

$$
\gamma=900 \mathrm{~kg} \mathrm{~m}^{3}, c_{s}=0.5 \mathrm{kca} / \mathrm{kg}, \Delta T \text { is } 0.000834 \mathrm{p}
$$

so that as P decreases it drops from $60^{\circ}$ to about $2^{\circ} \mathrm{C}$ at the end of the catapult process, as shown in Fig. 55. If we assume an external temperature of $+15^{\circ} \mathrm{C}$, then at the end of the tow path a lubricant is required which has a riscosity of $\%=0.00002$ at $+17^{\circ} \mathrm{C}$; e.s.; benziae water, petrolem; three meters from the beginning of the takeoff we need a labricant withof $=0.1$ at $75^{\circ} \mathrm{C}_{i}$ this requirement is satisfied by pitch. In between there is a whole range of possible lubricants. In the first three meters of the slide, pure liquid lubrication will not be possible; one will have to use graphite-pitch mixtures. The equations given are valid only for moderate sliding speeds; actually the velocity of motion reaches a final value of $500 \mathrm{~m} / \mathrm{sec}$ which is $1 / 3$ the velocity of sound in the lubricant; because of this we may expect a great incrane in the coefficient of friction, but no physically different friction phatomena, enpecialiy since the high velocities are accompanied by small pressure on the sliding jaws. The proper lubrication of the 3 km long surface should be maintained and protected by a special lubricant carriage which runs the full length of the track, along the rails. The brake arrangement at point B can be modelled on that of a railroad; i.e., cast-iron jaws are préssed onto the braking surfaces of the rail; after the aixcraft rises the forces are released and the empty sled is brought back to rest as quichly as possible.

To test the basic feasibility of sliding at high speeds, A.Schmid has made tests on the sliding of projectiles along curved metal walls lubricated with vaseline. The experimental arrangement consisted of a drawn-steel tube of 10 mm inner diameter, which was slit along its axis to produce a U -shaped groove. This channel was bent into a closed circle 8 m . in radius, so that the surface of the channel was toward the outside; then the projectile sliding in the channel is kept there by centrifugal force. Through a branch tube which was tangent to this circle, an ss projectile was fired from a German $98 \mathrm{~K}_{\text {; }}$ and into the circular path. The bullet ran through the circular path several times till it was brought fo rest; after the test it was found lying completely unharmed in the channel (Figs. 56, 57, 58) - The thin copper plating of the steel was, in mast tests, rubbed off at the position of the maximum diameter; the steel surface showed isolated scratches apparently caused by filings and burrs along the slide-path. Immediately after finding it, the bullet was atill warm to the touch. Also it was conpressed in cross-section; this is probably cansed by the fact that the spinning of the bullet stops after a short time and then the centrifugal force, which is initially 100 kg , causes the lead core to flow. After cessation of rotation and slight defornation, the initial point contact with the wall is increased to a surface of about $1 \mathrm{~cm}^{2}$, so that the $100 \mathrm{~kg} / \mathrm{cm}^{2}$ pressure between the sur faces, which is caused by the centrifugal force, corresponds approximately to that of the sliding jaws af the takeoff sled. Since the ss -bullet is made of lead with a thin soft steel cover, and in the course of its 150 m . sliding path goes through all velocities from $800 \mathrm{~m} / \mathrm{asc}$ down to zero without being harmed, this experiment may be considered as a proof of the feasibility of construction of sliding jaws for velocities of $500 \mathrm{~m} / \mathrm{sec}$; provided the slide-rails are properly canstructed and lubricated.

If the accelerations of the aircraft during its launching are to remain within permissible limits for both aircraft and pilot, a launching path of about 3 km . is required, so that it is not possible to use a movable construction which would point the craft topard its target at takeoff. One will therefore install takeoff track, usable in both directions, so that it points in the most probable directions for atack, say east-to-west; the more precise correc. tion of the direction of flight can be left to the pilot during the very first part of the climb path, where the velocity and acceleration are still sufficientiy small to permit such changes of path.

If the homber rises off the takeoff $-s$ led at $500 \mathrm{~m} / \mathrm{sec}$, then its lift coefficient is $c_{a}=$ 0.05 , whereas the best glide-number occurs at nuch larger angles of attack and a lift coefficient $c_{a}=0.173$. If one limits the normal acceleration during the first driveless part of the climb-path to $20 \mathrm{~m} / \mathrm{sec}^{2}$, then the initial radius of the turning-path, which goes frod the horizontal takeoff direction to the direction of climb at $\varphi=30^{\circ}$, is 12500 m . This curved path of tarn, which begins at the end of the takeoff is 5540 m . long and ends at an altitude of 1700 m. , with the required path inclination of $30^{\circ}$ As a result of air resistance and gain in altitude, the velocity at this point has dropped to $370 \mathrm{~m} / \mathrm{sec}$, the lift coefficient is $\mathrm{c}_{\mathrm{a}}=$ 0.093 - still mach smaller than the one for optimum glide-number. The bomber then climbs further at an angle of $30^{\circ}$, at the expense of its flying speed until, after a linear climb of 4000 m . to a height of 3675 m . and velocity $V=284 \mathrm{~m} / \mathrm{sec}$, the angle of attack for optimum glide-number is reached; at this moment the rocket drive for the actual climb can begin. During: this 25 -second-long, undriven motion after takeoff, the path of the aircraft can be turned in the direction of the target. At the same time, this procedure helps increase the range of the bomber ovar its value if one took of at $V=250 \mathrm{~m} / \mathrm{sec}$ تith optimum glide-number and started the engine imnediately after takeoff, because we have gained a slight excess velocity and a few kilometers for the clinb.
NThese Figs. omited


Fig. 55; Gliding velocity $v$, moved load $G$, loading on the runners $G-A$, heating of the lubricating film $\Delta T$, viscosity $\geqslant$ of the heated lubricating agent, and practical manner of Iubricating for various points along the starting path.


Abb. 56; Verguchanlage firr Gleitreibungsversuche bel sehr hohen Gleitgeachwindigkeiten, beatehend aus einem Militarkarabiner 98x und einer mit 8 a Radius gekrumiten, geschmierten und geschlossen-kreisformigen Gleitbahn ftur das sS-Geschoss.

## 3. Climb Path

The climb path of the rocket bomber is determined by the forces acting on its center mass. If one momentarily neglects the rotation of the earth, these are;

The weight of the aircraft-magnitude $G=\left\langle G_{0}-P_{g} t /</ / R /(R+\mu]^{2}\right.$ direction toward the center of the earth.

The aerodynamic lift-magnitude $A=c_{0} F_{\rho} \nu^{2} / 2 / 2 \quad$, direction perpendicular to the tangent' to the path and in a plane through the center of the earth;

The air resistance-magaitude $W=E A$ in the direction of the tangent to the path;
The thrust of the motor-magnitude $P=100$ toas, tilted fron the tangent to the path by angle of attack, lying in the plane through the center of the earth and the tangent to the path-

The $d^{\prime}$ Alembertian inertial force $T$, equal and opposite to the resultant of the other forces, having a tangential component $m \frac{d y}{d t}$ and a normal component on $\mathrm{Va}^{2} / \rho$.

These five forces all lie in a plane, so that the path is also in a plane.
The rotation of the earth makes the situation more complicated. If the earth's atmosphere were fixed in space, so that it did not rotate atong with the earth, then the path relative to the earth's surface could the calculated as if a sixth force, the Coriolis force, having arbitrary direction and magnitude $C=2(\vec{\omega} \cdot \psi(G / / \rho-F / C)$ acted on the center of wass of the aircraft. Because of its arbitrary direction in space, this force makes the prbit, relative to an observer on the earth, appear to be no longer in a plane. The atnosphere, which actually rotates with the earth, produces dragging forces on the bomber which canse its absolute path also to be twisted, so that the bomber is pulied like a weather vane by the apparent wind developed by the ratation of the atmosphere, and will tend to turn except when it is flying in the west-east direction, as can be seen by considering flight starting from the pole. This interfering weathervane action will have to be countered by the pilat's steering; i.e., by a seventh force perpendicular to the tangent, if the absolute path prescribed by calestial navigation is to be maintained. In the calculation, it is assumed that the transverse aerodynamic forces excited by the elimination of the weather-vane action do not noticeably alter the glide-number. Since these transverse forces are alwaye perpendicular to the plane of the orbit and only prevent a shift of the aircraft out of the plane, they need no longer be considered in the calculation of the orbit. To illustrate the the relation between the forces acting on the center of mass, Fig. 59 shows two views of the aircraft during the climb the line of sight is along the horizon at the level of the aircraft. If all the dependences were available in analytic form, one could introduce suitable space coordinates, resolve the forces along these directions, apply the dynamical equation to each direction and thus obtain three differential equations for the three cnordinates of the climb path; the integration would almost certainly be impossible, since the far simpler equations of ordinary exterior ballistics are not. The determination of the relative orbit is much simpler; one proceads in two steps; first the absolute path is caleulatea neglecting the Coriolis force (i.e., using the five forces previously listed). Then one calculates separately the turning of the earth's surfaco below the aircraft. Finally, the two components are combined on the sphere to give the twisted relative path in space. The plane absolate orbit is detenained by a step-by-step method which is familiar in Ballistics. ( $1, V_{0} .1, \mathrm{p}, 207$ ) As in Fig. 60 , the continuonsly curved path is broken up into a polygon, whose sides are sufficiently short that they can replace the arc of which they are the chord. All the forces acting on the aircraft are combined at the vertices, A, $B, C$ etc., and along with the inclination $\boldsymbol{o}$ of the orbit are assumed constant along $\Delta s=\overline{A B}, \overrightarrow{B C}$, et. By setting the resultant force in the tangential direction equal to zero, we obtain $\sigma_{j}=9\left(P_{c o s e x}-\in A_{A}-G_{A} \sin \varphi_{A}\right) / G_{A}$
to that the velocity increases in first approximation from $V_{B}$ to $V_{B /}=\sqrt{V_{A} Z_{2}} 2 \angle A S$ in the time $\Delta t_{1}=V_{0}-1 / b_{0}$. By setting the resultant normal force equal to yero we
 can be determined with sufficient accuracy fram the velocity of fight $V_{A}$ and the angular velocity of the point of takeoff. Thus the arc $\overparen{A B}$ can be drawn, and $B$, is determined in first approxioration. Now that the average value of the inclination and the average force between $A$ and $B$, are know, one can locate $B$ in second approximotion in the same manner. One repeats the procedure at B, often enough so that the orbit is given in analytical or graphical form. The calculations of the orbit by this method were done numerically by A Woyczechowshi; the initial conditions were beight 3675 m . above sea levei, relative velocity of flight $284 \mathrm{~m} / \mathrm{sec}$, and inclifation of orbit $30^{\circ}$, and thppath was studied out to $G / g_{0}=0.1$. The orbit was also calculated for $\mathcal{E}=3000$. 4000 and $5000 \mathrm{~m} / \mathrm{sec}$ and finally neglecting the earth's rotation, and then with maximum effect of the


F1g. 59; External forces on the Rocket Bomber in climbing.

Fig. 60; Approximate graphical solution for the climb path

|  |  |  |  |  | $\begin{array}{\|ll\|} \hline 1 & 5 \\ 0 & 5 \\ \hline & 5 \\ \hline \end{array}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { 券 } \\ & \text { E } \\ & \text { S } \\ & \text { 2 } \end{aligned}$ | 1000 | 0,60 | 50,0 | 37,0 | 55 | +15,5 | 12,8 | $-6,8$ | 124 |
|  | 2000 | 0,42 | 31,8 | 46,5 | 135 | +0,4 | 22,0 | -6.7 | 178 |
|  | 3000 | 0,30 | 20,0 | 41,5 | 230 | -4,1 | 30,4 | -4,8 | 214 |
|  | 4000 | 0,22 | 11,5 | 36 | 320 | -3,7 | 410 | +3,7 | 240 |
|  | 5000 | 0,15 | 4,8 | 34,5 | 405 | +1,2 | 558 | +370 | 261 |
|  | 6000 | 0,10 | 0,3 | 40 | 485 | + 7,3 | 77, ${ }^{\text {a }}$ | +39,1 | 274 |
|  | 1000 | 0,69 | 58,7 | 33,5 | 00 | +8,2 | 12,1 | -7, | 124 |
|  | 2000 | 0,53 | 4.3 | 31 | 150 | $-8,1$ | 19,1 | -5,5 | 189 |
|  | 3000 | 0,40 | 30,5 | 44,5 | 280 | +7,0 | 18,3 | $+26,8$ | 243 |
|  | 4000 | 9,30 | 20,0 | 47 | 425 | +9,5 | 28.3 | - 2,4 | 283 |
|  | 6000 | 0,23 | 13,3 | 68,5 | 560 | +8,1 | 37,7 | $-4,0$ | 313 |
|  | 6000 | 0,18 | 8,0 | 84 | 675 | +7,4 | 47,5 | -3,0 | 333 |
|  | 7000 | 0,14 | 3,8 | 98,2 | 785 | +7,3 | 61,3 | -0,8 | 350 |
|  | 8000 | 0,11 | 1,0 | 109 | 860 | $+7,6$ | 81,6 | +1,6 | 362 |
|  | 1000 | 0,75 | 65,0 | 31 | 60 | +3,1 | 11,5 | $-7,3$ | 128 |
|  | 2000 | 0,52 | 54,7 | 18,5 | 155 | -10,6 | 16,3 | +5,0 | 196 |
|  | 3000 | 0.48 | 37,5 | 43,5 | 335 | +11,9 | 16,7 | -3,9 | 267 |
|  | 4000 | 0,38 | 28,1 | 71 | 505 | +7,3 | 22,3 | $-6,3$ | 315 |
|  | 5000 | 0,30 | 20,1 | 87,5 | 665 | +4,4 | 28,8 | -5,8 | 351 |
|  | 6000 | 0,25 | 15,0 | 98,5 | 830 | +2,9 | 348 | -5,1 | 380 |
|  | 7000 | 0,20 | 10,5 | 106 | 975 | +2,6 | 41,0 | $-4,1$ | 404 |
|  | 8000 | 0,7 | 6,5 | 112,3 | 1135 | +1,8 | 49,5 | -2,9 | 424 |
|  | 9000 | 0,14 | \$,5 | 117 | 1275 | +2,0 | 693 | -1,3 | 440 |
|  | 10000 | 0,11 | 1,0 | 121,5 | 1400 | +2,3 | 73,3 | 0,4 | 452 |

[^3]

Absolute climbing path of the Rocket Bomber with $c=3000 \mathrm{~m} / \mathrm{sec}$, without consideration of the Earth's rotation, bat with consideration of rotation velocity of a point on Earth's equator, launch to East and launch to West.


Fig. 62: Absclute climbing path of the Rockst Bomber with $c=4000 \mathrm{~m} / \mathrm{sec}$, without consideration of the Earth's rotation, but with considaration of rotation valocity, of a point on Earth's equator, launch to East and launch to West.


P1g. 63:
Absoluts climbing path of the Rocket Bomber with $c=5000 \mathrm{~m} / \mathrm{sec}$, without consideration of the Earth's rotation, but with consideration of rotation valocity, of a polnt on Earth's equator, launch to East and launch to West.


Fig. 6:4 The absolute filight velocity of the Rocket Bomber attained during climb with $c=3000,4000$, and $5000 \mathrm{~m} / \mathrm{sec}$ as a function of $G / G_{0}$, and without considaration of Farth's rotation.
rotation - d. e., takeoff from the equator in an easterly or westerly direction. The following figures show some examples of the rexults; Fig. $61-63$ show the shape of the absolute orbit, Fig. 64 shows the velocities reached. Table III eives a sumary of the nost inportant results for all the orbits, neglecting the earth's rotation.

A fairly accurate estimate of the final velocity, which is the most important datum for the climb path, can be obtained without doing the exact calculation of the orbit, by using the equation $v / c=C_{\pi} G_{0} / G-q\left(1-G / C_{0}\right)$ This is especially so if one fits the actual conditions by setting $q$ (ratio of total air resistance plus weight component along the path to instantaneous weight) equal to 0.5 in the range $G^{\prime} G_{0}=1.0$ to 0.5 , and $q=0$ for $G / G o=0.5$ to 0.1 . If one carries out this rough calculation for the bomber with 100 ton takeoff-weight and 10 tons empt, weight, for various exhaust speeds C, assuming initial velocity $284 \mathrm{~m} / \mathrm{sec}$, one finds that for $C$ $=4000 \mathrm{ny}$ sec the borimr reaches the velocity of a long-range projectile even with a 50 -ton bomb load, so that it can remper with long-range artillery; without payload it exceeds the velocity $\mathrm{V}=7900 \mathrm{~m}$ 'sec. Betufen thess two extremes lie all sorts of velocities and bomb loads, An interesting result which is easily obtained, is that in the range of bomb loads suitable for military use, i.e. 5-40 tuns, the exhaust speed has little effect on the final speed. This means that the development of a rocket bomber docs not have to wait for availability of rocket motors with $C=4000 \mathrm{~m} / \mathrm{sec}$; with $\mathrm{C}=3000 \mathrm{~m} / \mathrm{sec}$ one can already construct a very dangerous weapon.

Figs. 65 and 66 compare this rough calculation to exact orbit calculation and to the curve for ideal rocket propulsion; the initisl velocity efter takeoff is included. One sees that the rough estimate gives the actual behavior with sufficient accuracy, especially for large values of $\mathrm{C}_{\text {; }}$ also the final velocities actually attained, especially with large bomb loads, are far below those for ideal rucket propulsion, because tho weight component in the direction of propulsion is comparable to the thrust for the initial steep climb, and because of the rather high air resistance during the climb. The propulsion curve could thus be improved by lessening the resistance or increasing the propulsive fortes. A lowering of the initial nngle of climb $f$ would decrease the harmful weiglit component, but would result in great increase in air resistance in regions of detse air - we can expect no success from this procedure. Another possibility might be a great increase in the cisrust during the initial part of the path, so that, say, the tangential accel: ipetion is constant in the range of the pemissible limit 10 g , and the resisting forces are half the instantaneous weight initially and then drup to zero below $\mathrm{G} / \mathrm{G}_{0}=0.5$. The rocket motor would have to start with a thrust of 1000 tons, and then be throttled gradually to 100 tons at the end of the climb. Taking $v=284+095 c l_{0} G_{0} / G$ for $10>G / G_{0}>0.5$ and $v=284+0.59+$ $c \ln G_{0} / 2 G$ for $0.5>G / G_{0}>0.1$ we get the results shown in Fig. 67. While the $100-\operatorname{ton}$ rocket motor weighs 2500 kg ; one must assume a weight five times as great (i.e., 12500 kg ) for a well-regulated motor with 10 times greater maximum thrust. If, despite the more powerful motor, all other weights of the bomber remain the same; then the empty weight would be 20000 kg . instead of 10000 kg ; i.e., the most favorable $G / G$ would bg 0.2 , and the bomb load would already have dropped to zero at $G_{o} ; G=5$. The propulsion curve shown in Fig. 6.7 for 10 -ton empty weight would, for the now necessary 20 -tons, move considerably to the right; despite the increased difficulties with the regulable 1000 -ton miotor one would achieve only a slight improvement for short distances of attack, while the 100 -ton motor would still be superior for long range attacks. Thus the-second method for improving the propulsion curve also fails in practice, and the curver of climb shown in Figs. 65 and 66 represett the best solution of the problem.

The rocket bomber, at each moment of its flight, tends toward that part of the atmosphere where the buoyant forces are equal to its weight. If this equilibrium position is such that the altitude stays constant for a few seconds, it will be said to be stationary, Such a stationary equilibrium occurs, for example, if the aerodyamic driving force A of the bomber, increased by the centrifugal force $F_{1}$ which for flight at constant altitude is a consequence of the earth's curvature, is equal to the instantaneous weight, $G$ of the bomber.

Luring the climb the thrust component Dsinc also acts as a buoyant farce. For example, starting with a weight in flight, $\mathrm{G}_{0}=100$ tons, at altitude $\mathrm{H}_{\mathrm{o}}=3500$ m, and $\mathrm{C}_{\mathrm{a}}=0.2$, by equating the weight to the propulsive force, we can obtain for the horizontal velocity a value $v_{0}=\sqrt{e_{g} G_{0} / c_{a 0} \delta^{2}}=500 \mathrm{~m} / \mathrm{s}^{2} c$. At any other altitude, if we set the weight equal to the buoyant force $c_{a} \delta^{2} \nu^{2} F / 2 g+G V^{2} / \sigma^{2}=G_{w e}$ get a one-to-one relation between altitude and velocity. This relation is shown in Fig. 68 , for $\mathrm{C}=4000 \mathrm{~m} / \mathrm{sec}$. We see that these stationary altitudes are law, below 60 km for velocities up to $7000 \mathrm{~m} / \mathrm{sec}$. If we include the vertical component; Poines of the motor thirust of 13920 kg , we get the upper curve. It is interesting to note that for $\mathrm{V}=$ $4700 \mathrm{~m} / \mathrm{sec}$. the curve becomes vertical; i.e., no stationary equilibrium is possible at finite altitude as soon as $\mathrm{V}=4700 \mathrm{~m} / \mathrm{sec}$. In the first curve the stationary equilibrium is no longer attainable above $V_{-} 7900 \mathrm{~m} / \mathrm{sec}$. These assumed stationary states probably do not exist in flight. In particular, the center of curvature of the path is seldom at the center of the earth so that the altitude would remain constant - most of the time it is near the earth's surface, and at times it is even above the sircraft also the radius of curvature is usually much less than the radius of the earth. As a result strong transverse dynanical forces are developed, which permit "dynamical" equilibrium of the bomber at heights far above the stationary ones.

Bomb load in tont


Fig, 65:
More exact determination of the relationship between tomb load, terminal velocity of the Rocket Bomber attained for $v=3000 \mathrm{~m} / \mathrm{sec}$ and $\nabla_{0}=284 \mathrm{~m} / \mathrm{sec}$ by means of the graphical determination of the course. For comparison are plotted the approximate calculations and the curve for ideal Rocket acceleration.

Bomb load in tons


Pig. 66:
Nore axsct determination of the relationship betreen bomb load, teminal velocity of the Rocket Bomber attained for $\nabla=4000 \mathrm{~m} / \mathrm{gec}$ and $v_{0}=284 \mathrm{~m} / \mathrm{sec}$ by meana of the graphical detarmination of the ccurse. For comparison are plotted the approimate calculations and the curve for idea. Rocket acceleration.


Fig. 67:
Climb of a Rocket Bomber whose thrust is adjustable between 100 and 1000 tons, i.e, high almost constant accoleration with $c=4000 \mathrm{~m} / \mathrm{sec}$ axhaust velocity.

The actual heights oscillate about the stationary ones, with very large amplitude, while the additional dynamical forces tend to bring the aircraft back to the position of stationary equilib. rium; i.e., they are restoring forces pointing toward the stationary position and increasing with distance of the aircraft fromit, like spring-forces. The number of oscillations; for the example shown in Fig. 68, is not quite 2 for the whole climb; this is shown by the following consideration.

At first, the stationary altyitude is greatly exceeded. This results from the fact that initially the air density decreases more slowiy than the increase in velocity, so that the aircraft takes on a very steep position at angles $\boldsymbol{\varphi}=45^{\circ}$ or more. Already at fairly high velocities, the aircraft must now be turned through more than $45^{\circ}$ from this position to that of horizontal flight. Because of the small radius of curvature available, this results in powerful centrifugal forces, which drive the aircraft up above the stationary equilibrium heights of $40-60$ km , to heights over 100 km . (me might now expect that the aircraft would imnediately drop down again from these extreme altitudes, since the path is approaching parallelism with the earth's surface, and the radius of curvature is approaching the value of the earth's radius. This drop from the heights initially attained occurs only in slight degree, if at all, because in the meantime the velocity has increased very rapidly, and is already near the value $V=4700 \mathrm{~m} / \mathrm{sec}$ for which the altitude of stationary equilibrium of the bomber becomes infinite. At this speed, the bonber is in stationary equilibrium at every altitude, so that it daes not have to fall. If it exceeds $V=4700 \mathrm{~m} / \mathrm{sec}$, then it suddenly finds itself below the altitude of stationary equilib. rium, so that it has an excess of buoyant force, and begins to climb again so long as the propulsion continues. This actual curve of climb is obtained from the exact calculation of the orbit, and is shown in Fig. 68. The whole climb may be considered as dynamic flight only in a limited sense. For the most part it follows an inertial path, like that of a projectile or a heavenly body, at heights which are practically in empty space, since one can no longer speak of an atmosphere in the sense of aerodynamics when the molecular free paths are greater than several hundred meters. Fig. 69, obtained from exact orbit calculations, shows the actual dynamical altitudes attained during the climb of the rocket bonber.


Fig. 68:
Comparison of stationary (i.e. instantaneous equilibrium) and actual, dynamic flight altitude when the Rocket Bomber is clinding with $c=4000 \mathrm{~m} / \mathrm{sec}$.
s/u000s-2


[^4]
## iV. GLIDING FLIGHT AND LANDING

## 1. Supersonic Path of Gliding Flight:

The supersonic path of gliding flight of the rocket bomper is determined by the forces meting on its center of mass, in the same manner as for the climb path. These external forces are: the weight of the aircraft, $G=\operatorname{mg}_{a}(\mathrm{~F} / \mathrm{R}+\mathrm{H}){ }_{2}$, the aerodynamic lift $A=\mathrm{C}_{4} \mathcal{F} \mathrm{p} / 2 / 2$, the air resistance $W=E A$, and the $d$ 'Alembertian inertial force $T$. The same results apply to the rotation of the earth and the atmoaphere as in Sec. III 3; in particular, the supersonic gliding path is to be considered as occurring in a plane, for an observer out in space; the deviating effect due to atmospheric ratation, which is especially effective in the lover layers of the atmosphexe, must be eliminated by stetring. Only in special cases will the Coriolis force be eliminated by steering, so that the orbit relative to the earth is plane; this will oceur if the bomber is to go all around the earth on one hop, and land at its starting point. To do this in a perfectly plane absolute orbit is possible only if the takeoff field is at the pole, or if it is at the equator and the plane of the orbit coincides with the equatorial plane. Otherwise the procedure of circumnavigation will go as follows: until the bonb release an absclute plane orbit will be flown for purposes of precise celestial navigation; only the weather-vane action will be eliminated. After the bomb release a suitable relatively plane orbit will be flown, i.e., both weather-vane and Coriolis effects will be eliminated by steering. The arrangemeat of forces used in calculating the path are shown in Fig. 70 for gliding flight in two aspects, both viewed along the horizontal at the level of the aircraft. The procedure of calculation corresponda exactly to the stepwise method used in determining the clinb path; first the absolute orbit is determined neglecting the Coriolis force and then the separately computed rotation of the earth is combined with the absolute orbit to give the desired relative orbit. The initial conditions far the decending path are given by the endpoint of the climb path. Since the climb path can be broken off at any point, there is a singly infinite manifold of possible paths of descent for each ascending path. Actualiy we started from the previously calculated ascending paths with $c$ $=3,4$ and $5000 \mathrm{~m} / \mathrm{sec}$. and each of these was hroken off at $V_{0}=1,2,3 \mathrm{p}$ to $8000 \mathrm{~m} / \mathrm{sec}$, provided that all these velocities were actually reached. They represent the initial velocities for the descending path. Each descendiag path is followed until the velocity has dropped to $300 \mathrm{~m} / \mathrm{sec}$. In Fizs. 71 and 72 two paths are shown; one for $c=3000 \mathrm{~m} / \mathrm{atc}, V_{0}=4000 \mathrm{~m} / \mathrm{sec}$; the other for $C$ $=3000 \mathrm{~m} / \mathrm{sec}, V_{o}=6000 \mathrm{k} / \mathrm{sec}$ at 6.36 times the altitude. In the figures, the strong oscillations of the paths, especially in the first portion, are notable. Because of the considerable talt of the orbit resulting from the climb, the bomber overshoots its stationary altitude of flight, then approaches it from above, passes through it because of inertia, then is driven upward again by the grestly increased aeradynamic forces, until after several such oscillations the amplitude has decreased so much that the airctaft levels off at the stationary altitude and continues its flight at that altitude. This ricocheting generally has favorable effect on the range of the bomber. It has the advantage that the themal stresses of the external surfaces of the aircraft which face the course wind vary in time at high velocities of flight. The oscillation of the path will be bindered by steeriag, only if fight at the stationary altitude is needed for some special reason, say aiming before the bomb release.

Figs. 73 and 74 show the main elenents of the two orbits in greater detail. In Fig. 74, one can easily see the following connections: the orbit shown in Fig. 72 is now represented by ploting the altitude 1 against the $\log$ of the distance $S$. One can easily locate the peaks and valleys of the orbit for both curves. The climb path ends after $t=300 \mathrm{sec}$. at an alcitude $\mathrm{H}=41.2 \mathrm{~km}$, with inclination $90=6.20$. At this moment the velocity $\mathrm{V}_{0}$ reaches $6000 \mathrm{~m} / \mathrm{sec}$. When the rocket motor is turned of fine tangential acceleration drops from ${ }^{0} b_{t}=+75.5 \mathrm{~m} / \mathrm{sec}^{4}$ to $-5.5 \mathrm{~m} / \mathrm{sec}^{2}$, so the aircraft is greatly retarded, considering the flat ascending path. At the
 the normal component of the engine thrust has ceased, to $b_{n} \neq+18 \mathrm{~m} / \mathrm{sec}^{2}$. This positive residue results from the instantaneous center of curvature being above the aircraft, In the first part of its motorless supersonic glide path, the bomber climbs to $\mathrm{H}=143.8 \mathrm{~km}$, while the ivelination of the path goes through maximum $y^{\prime}=+$ afiliso the tangential acceleration gradually increases to by 0 o at the crest of the wave, while the nomal acceleration increases up to $b_{n}=-10 \mathrm{~m} / \mathrm{sec}^{2}$, so that over a long period of time objects in the aircrait will appear to be weightless. At the same tine the velocity drops to a minimum of $V=5800 \mathrm{~m} / \mathrm{sec}$ at the crest of the wave. At the position, $S=1700 \mathrm{~km}$, of the first crest in the orbit, the curves for $\mathscr{P}$ and dso practically for $b_{t}$ (because of the low air-density) must go to zero, while $6_{n}$ and $V$ pass through a minimstr. Then follows the first flight into a valley, with a height loss of 108.8 kn. in 250 sec . Luring this the velocity again rises to $6000 \mathrm{~m} / \mathrm{sec}$, the largest inclination is - $8^{\circ}$, the largest tangential acceleration is $6 t^{+} \equiv+/ m / \mathrm{mec}^{2}$. In the trough the height is $\mathrm{H}=35 \mathrm{kn}$, and the largest normal acceleration acting at the cockpit is $b_{n}=\neq 58 \mathrm{~m} / \mathrm{sec}^{2}$


Fig. 70: Fixternal forces on the Rocket Bomber in the ease of supersonic gliding.


Fig. 71: Flight path of the Rocket Bomber with $c=3000 \mathrm{~m} / \mathrm{sec}, \mathrm{r}_{0}=4000 \mathrm{~m} / \mathrm{sec}$ and a bomb load of 11.5 tons.


| $c$ | Vo in $\mathrm{m} / \mathrm{sec}$ | Bamb <br> load <br> in tone | Flight path 1ength in Kr | Time of flifat in | Maxifum flleght height in Kr | taximum positive normal acceleration En I/ $/ \mathrm{sec}$. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { पूँ } \\ & \text { N } \\ & \text { E } \\ & 0 \end{aligned}$ | 1000 | 50,0 | 303 | 490 | 40 | 38,5 |
|  | 2000 | 31,8 | 1528 | 1300 | 46 | 21,7 |
|  | 3000 | 200 | 3639 | 2180 | 45 | 17,4 |
|  | 4000 | 11,5 | 6692 | 2620 | 47 | 10,5 |
|  | 5000 | 4,8 | 12771 | 4330 | 76 | 34.9 |
|  | 6000 | 0,3 | 20371 | 5800 | 143 | 46,5 |
|  | 1000 | 58,7 | 295 | 530 | 34 | 28,7 |
|  | 2000 | 43,3 | 1367 | 1160 | 37 | 15,2 |
|  | 3000 | 30,5 | 3477 | 2100 | 49 | 26,0 |
|  | 4000 | 20,0 | 0959 | 3040 | 80 | 35,0 |
|  | 5000 | 13,3 | 12592 | 4400 | 104 | 45,3 |
|  | 6000 | 8,0 | 21139 | 5820 | 160 | 48,8 |
|  | 7000 | 3,8 | 39363 | 8840 | 283 | 50,3 |
|  | 8000 | 1,0 | 91870 | 16015 | 1296 | 50,7 |
| yEB.bin | 1000 | 65,0 | 291 | 455 | 30 | 22.2 |
|  | 2000 | 51,7 | 1254 | 1120 | 31 | 19,2 |
|  | 3000 | 37,5 | 3847 | 2225 | 68 | 37,3 |
|  | 4000 | 28,1 | 7454 | 3200 | 87 | 37,0 |
|  | 5000 | 20,1 | 12180 | 4290 | 102 | 46,5 |
|  | 6000 | 15,0 | 21531 | 5990 | 111 | 35,9 |
|  | 7000 | 10,5 | 42091 | 9120 | 128 | 36,3 |
|  | 8000 | 6,5 | 293720 | 41600 | 778 | 2,5 |

Numerical characteristics of 22 various descent paths of the Rocket Bomber.




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This procedure is repeated at aach juinp which the rockec danbes .,wnes laward ifs target, while the extreme values gradually die down and the flight becomes more and more smooth in the neighborhood of the stationary altitude. The total length of the projection of the glide path on the earth's surface is 20371 km . the duration of the flight is 5830 sec . In this manner various orbits were calculated; Table II gives the most important data - length of the orbit, glide. number, maximum altitude attained, and maximum positive noxmal acceleration.

All these orbit investigations were carried out neglecting the earth's rotation. For this reason Figs. 75 and 76 stow the relative and absolute orbits for supersonic flight of the rocket bomber for $c=3000 \mathrm{~m} / \mathrm{aec}$ and with the velocity at the end of the climb path $\mathrm{V}_{\mathrm{o}}=6000$ $\mathrm{m} / \mathrm{sec}$, if the aircraft takes off in an exact easterly or westerly dizection from a point on the squator. In the case of an eastward takeoff, the velocity of rotation of the takeoff point relative to the earth's centex adds to the velocity of the aircraft relative to the takeoff point. so the centrifugal force increases. Because of this influence of the earth's rotation, the range $S=20370 \mathrm{~km}$. in Fig. 72 is increased to $S=23470 \mathrm{~km}$, nore than $13 \%$. This supersonic path relative to the earth's surface is show in Fig. 75. The most interesting features are the much greater heights of the orbit peaks and the longer distance between peaks, compared ta Fig. 72. The absolute path of the aircraft, as seen by an observer out in space, is shown doted in Fig. 75; naturally the absolute path length is much greater ( $S=26410$ kfti). In the case of takeoff to westward, the rotational velocity of the takeoff point is subtracted fron the velocity of flight, the centrifugal force decreases and the range drops from 20370 km to 18200 km , i.e., more than $10 \%$. At the same time the heights of, and intervals betwetn, the first waves of the orbit detreast. These results are shown in Fig. 76 ; the absolute orbit is shown dotted for comparison. The effect of the earth's rotation on the range and height of the supersonic orbit becomes even greater if the maximum velocity of flight approaches the velocity due to the earth's rotation; on the other hand it decreases rapidly if the start is made from the equator in an intermediate, rather than in an easterly or westerly direction, or if the takeaff point moves from the equator toward the poles. For example, the ranges for $c=4000 \mathrm{~m} / \mathrm{sec}$ and $V_{p}=7000$ $\mathrm{m} /$ sec are 32430 km . for westward takeoff from the equator, 50440 km for eastward takeoff from the equator, and 39363 km for takeoff fron the pole; the corresponding rangen for $\epsilon=5000$ $\mathrm{m} / \mathrm{sec}, \mathrm{V}_{\mathrm{o}}=7000 \mathrm{~m} / \mathrm{sec}$ are $32660 \mathrm{~km}, 58880 \mathrm{~km}$ and 42091 km .

When the bonb load is dropped during the supersonic gliding flight, the weight suddenly decreases by 10 tons from the value $G$, and the stationary altitude increases $b$; $\triangle H=634 / G G / 0$. This decrease and the diminished ballistic loading of the aircraft produce an effect on the oscillating orbit, as shown in Fig. 77 for a definite case. It was assumed that, along the orbit for $c=4000 \mathrm{~m} / \mathrm{sec}, V_{0}=7000 \mathrm{~m} / \mathrm{sec}$, a 3.8 ton bomb was released harizontally at 40 km altitude and $6060 \mathrm{~m} / \mathrm{sec}$ velocity, so that it struck the point on the earth opposite to the takeoff point. For a bothb throw of 850 km , the release must oceur after 19150 km . At this point the plot of the orbit splits into three curves; the path of the falling bomb, che dotted path which the bomber would have followed if no bombs were released, and the solid line showing the orbit after the bomb reJease. In the last case the waves in the orbit are higher and broader, so that after several oscillations a definite phase shift is observable, the stationary part of the path lies 1670 m. higher, and the final velacity of $300 \mathrm{~m} / \mathrm{sec} 1 \mathrm{~s}$ reached a few dozen kilometers sooner. The difference in range is so slight that one need not make a special investigation of the orbit after the bonb release, but can use the approximate orbit calculated for fuls bamb load. Fig. 78 shows the elements of this orbit; it is interesting, to note that at the point of release the normal. acceleration jumps discontinuously from $+7 \mathrm{~m} / \mathrm{sec}^{2}$ to $+19.5 \mathrm{~m} / \mathrm{sec}^{2}$, because the aircraft was in a trough and before the bomb release (when it weighed 13.8 tons) it was in fynamical equilibrium with the buoyant forces of the air.

The range of the rocket bomber is largely determined by the length of the supersonic glide path. This important quantity can be estimated to a first approximation, without doing the exact orbit calculation, by setting the inertial force equal to the air resistance, G/God/r/ = -GE from which $v=1 / E_{g} t$ and $s=\left(v^{2}-v / 2 \varepsilon_{9}\right.$

This simple calculation is satisfactory up to $V_{e}=2000 \mathrm{~m} / \mathrm{sec}$. Above this, the centrifugal force due to the curvature of the bomber s orbit around the earth can no longer be

$$
t=\frac{p}{2 \epsilon \sqrt{R_{g}}} \ln \frac{\left(\sqrt{f_{g}}+r_{g}\right)\left(\sqrt{\sqrt{x}_{g}}-r\right)}{(\sqrt{\sqrt{g}-1}-1)(\sqrt{\sqrt{g}}+v)} ;
$$

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Fig. 79: Comparison of the ranges of the supersonic descent path for equal initial velocities computed by
a) Constant flight altitude braphical deternined flight aititude with $c=3000,4000$ and
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 Fiight length in km

Fig. 80: Relationship between load ratio $\lambda=\mathrm{G} / \mathrm{G}_{\mathrm{O}}$, bomb load B , and range of
the Rocket Bomber for a "straight ahead path" and for exhaust veloaities
of $\mathrm{c}=3000,4000$ and $5000 \mathrm{~m} / \mathrm{sec}$.

These equations give good results for the atationary flight patis until we approach the velocity of points on the earth's surface. With the aid of these equations we can also include the effect of the earth's rotation, and obtain values of $V_{a}, t$, and $S_{a}$ for the absolute orbit, if in place of the relative velocity $V_{\text {we }}$ we the absolute ${ }^{\text {a }}$ velocity $V^{a}{ }^{0}$ of the takeoff point, which is calculated from its velocity $V_{e}$ and the compass angle of the initial velocity $V_{o}$ by the equation $V_{c i o}^{2}=\nu_{0}^{2}+V_{0}^{2}+2 K V_{e} \sin \xi$. The effect of the earth's rotation on the path length is greater than $1 \%$ in the most unfavorable case for $V_{0}=2500 \mathrm{~m} / \mathrm{sec}$, but decreases considerably when we change from absolute to relative path length. The path lengths calculated in this way can be compared to the ranges of fire for the same muzzle velocity $Y_{g}$; up to $V_{0}=6000 \mathrm{ol} / \mathrm{sec}$ and for the glide number $/ \in=6.4$ used here, they are better by a factor 3.4 , and as we approach the velocity of points on the earth's surface the factor increases rapidly; i.e, for the same initial velocity one can fly much farther than one can shoot.

Fig. 79 shows a comparison of the results of this second approximation with the stepwise calculation of the oscillating dyamical path. The reason why the dyamical paths are considerably longer than the stationaryfor the same initial speed is mainly that whereas in the stationary gliding flight the energy consumption is dittributed unifornly over the whole path, it is concentrated in the troughs of the dynamical path; the first (and also the longest) jump, which constitutes $15-30 \%$ of the total range, has a trough only at its end, and only loses there half of the energy which would be lost at the stationary altitude (this energy represents $15.30 \%$ of the total energy store). Other reasons for the greater range of the dynamical paths are that the regions of greatest energy consumption occur at low altitudes where favorable gas-dynamic glidenumbers at high Reynolds numbers exist, while the higher parts of the path, in which unfavorable gas-kinetic glide-numbers and low Reynolds nunbers exist, occur in regions of rarefied air or practically empty space; finally, the process of turning out of the climb path, in the case of the stationary orbit, has to oceur under unfavorable angles of attack, resulting in increased energy consumption for the same length of path. Even more clear is the plotiting of the supersonic range of flight against bomb load or weight, ratio, as in Fig. 80. The last two figures do not include the effect of the earth's rotation.

Also unportant for the use of the rocket bomber is the situation where, after release of Dombs, the aircraft starts toward its home base. If the antipodes of the home base or other parts of the opposite hemisphere are being attacked, the bomber will return simply by continuing along its glide path after the bomb release and flying all around the earth. For nearer targets, there is the possibility of return by shifting course after the bomb release. Threc possibilities are considered. First, instead of having the path be a great circle, after the initial propulsion, let it be a circle, lying on the earth's surface, with diameter Or equal to the distance between takeoff point and target. For flight along this circular cap, the air resistance for a

because of the oblique centrifugal forces, while the circular cap is shorter than the great circle only in the ratio $r / p$ Flight along the circular cap will be more favorable than flight along the great circle only for velocities below $5600 \mathrm{~m} / \mathrm{sec}$, so this method of turning seems suitable for shorter distances of attack. However, in this range there is in most cases another favorable turnang procedare - to reach the target along a great circle, to turn the aircraft at the target as sharply as possible, and return home along another great circle. if the bomber turns through a suall angle af, then the ratio of the work done, on the element of tura path of leagth $m$ to the kinetic energy of the bomber at the beginning of this element is $2 \in d=\sqrt{\prime \prime}-y^{2} / /^{2}$
which can drop to $h=V \neq 0$ because of large centrifugal acceleration which can approach the permissible limit. The turning path then becones a spiral along which the tangential and radial accelerations are constant. It is not integrable and must be computed step by step. From the equation, the relative loss of weight during the turn is independent of the velocity at the arart of the turn, so it can be computed once and for all for all the spirals. It is shown in Fig. 81 for turn-spirals with tangential deceleration $1.75 \mathrm{~m} / \mathrm{sec}^{2}$ and centripetal acceleration $50 \mathrm{~m} / \mathrm{sec}^{2}$. After a $90^{\circ}$ turn only $60 \%$ of the initial energy is left, after a $180^{\circ}$ turn only $37 \%$ is left; i.e., the sharp turn still casts a great deal of energy. There are intermediate precedures for turning, which are a combination of narrow turn with non-great circles: these are characterized by less energy consumption than the last limiting case which only pernits the use of oscillating flight paths.

A third turaing method consists in the aircraft using only enough energy in the climb, to enable it to reach the target; there it turns at its small residual speed, and with the aid of a fuel store on board, gets another push to give it the energy for the return home. This method of using two driving periods has the feature that the aircraft travels slowly over the target at low altitude. So on the one hand the bombs can be dropped with great accuracy on the other hand, the fire power is less than for long range attack, and finally the bomber gets into the enemy zone of defense at the target. Fig. 82 shows the ranges of the rocket bomber when it is turned by the last two methods.


Fig. 81: Loss in kenotic energy during the turning through an angle along the spiral turn. (see p. 122)



Fig. 83: Stationary flight altitudes of the Rocket Bomber during the supersonic glide with various bomb loads.


Fig. 84: Comparison of the actual and stationary flight altitudes of the Rocket Bomber during a supersonic glide path. (c = 4000 $\mathrm{m} / \mathrm{sec}, \mathbf{v}_{\mathrm{o}}=7000 \mathrm{~m} / \mathrm{sec}$.)

The same considerations apply to the altitudes of supersonic gliding flight as to the climb, especially as regards " stationary" and " dynamic "altitudes. The stationary altitudes, determined by the equilibrium between the constant flight-weight and the propulsive plus centrifugal forces (just as they were used for the second approximation to the length of the supersonic path), are shown in Fig. 83, where the curves are drawn solid oat to the point actually gitainoble with the given bomb load and $c=4000 \mathrm{~m} / \mathrm{sec}$. The dynamical altitudes of flight, which result from the varying initial conditions for the dynamical orbit of supersonic glide flight, tend to approach the stationary altitudes of flight, as shown in Fig. 84 which shows the dynamical and stationary paths for $c=4000 \mathrm{~m} / \mathrm{sec}$ and $V_{0}=7000 \mathrm{~m} / \mathrm{sec}$.

## 2. Path of Subsonic Gliding Flight and Landing

The subsonic gliding flight, by its definition, begins at $V=300 \mathrm{n} /$ sec with $\mathrm{G}=10$ tons. The lift coefficient for favorable glide position is then about $C_{a}=0.2$ and the corresponding height is $H=20 \mathrm{~km}$. The supply of potential and kinetic energy is still $24580 \mathrm{~kg} \mathrm{~m} / \mathrm{kg}$, with which a distance of $24580 / \varepsilon=98200 \mathrm{~m}$. can be travelled for an average subsonic glidenumber $E=4$. One can follow the descent in mall velocity-steps $V_{1}, V_{2}, V_{3}$. by using the result that the decrease in kinetic energy $\left(V^{2}-V_{2}^{2}\right) / 2 g$ plus that of the potential energy $\left(H_{1}-H_{2}\right)$ must always equal the work against air resistance $\Delta s . E$; in the stratosphere we get $\Delta s=\left(v_{r}^{2}-v_{2}^{2}\right) / 2 \varepsilon g+6341 / \varepsilon \ln v_{1}^{2} / v_{2}^{2}$ or $\Delta \neq 63 \mu / L_{1} V_{1}^{2} / v_{2}^{2} \quad$ in the troposphere $6.34 / 2$
 The subsonic glide path obtained from these equations is shown in Fig. 85. He see that the subsonic descent lasts 11 minutes, and ends near the surface of the earth at a velocity of $288 \mathrm{~km} / \mathrm{hr}$, whereupon the landing $c$ an occur. The actual variability of the subsonic glide-number can affect his path of descent.

The landing process begins at $V=288 \mathrm{~km} / \mathrm{h}$ with $\mathrm{C}_{\mathrm{a}}=0.2$. The behavior of the aerodynamic forces is given by the upper polar of Fig. 34, so that the velocity of the bomber can be lowered, by using landing-alds, to $288 \sqrt{2 / 2 / 0 x_{4}}=1.50 \mathrm{~km} / \mathrm{hr}$ which is required for military glide-landings. With these polers we can determine the air resistance and the required angle of attack $\propto \mathcal{C}$ for all velocities between 288 and $1.50 \mathrm{~km} / \mathrm{hr}$, for $\mathrm{G}=10$ tons, so the landing procedure can be followed by using the dynamical equations. It is shown in Fig. 86.



## - FHOJECTION OF THE BOMBS

## 1. Types of Projection

The rocket bomber has two different methods of attack: point attack and areal attack. The two procedures differ mainly in the manner and accuracy of the bomb release.

In the procedure of attack on a point, the bomb will be ained precisely toward a point"shaped individual target, and released at moderate altitade and velocity under the same conditions as for ordinary bonbers. In practice the same sub-types of bomb release are available to the rocket bomber as for other bombing aircraft, eg. bomb release during horizontal flight, dive bomb attack, bomb release during climb, low-altitude bonbing, etc. The well-known conditions and difficulties of these types of release apply practically unaltered to the rocket boanber, especially as regards the accuracy attainable and the need for adequate visibility at the target, so nothing new can be said about these typrs.

Things are quite different for attack on ar area. Here the bomb in thrown from great altitudes ( $50-150 \mathrm{~km}$ ) and at very high velocities of flight (up to $8000 \mathrm{~m} / \mathrm{sec}$ ), i.e. under con ditions far beyond those of long-range artillery fire. Since the target, for the distances involved, will not be visible, the release on an area will be aimed indirectly, e.g. by celestial navigation. Thus it is independent of weather and visibility at the target. Because of this, it does not reach the accuracy of release on a point, and we must expect spreads of several kilometers. So with areal bombing one cannot hit particular points, but rather a correspondingly large area, with sufficient probability. To achieve an anticipated effect on this whole surface, a single drop will not suffice, rather we will have to project several hombs toward the sane target; these will distribute themselves over the surrounding surface according to the lavs of chance. The distribution of hits inside the area will not be uniform; the bombs will strike more frequently in the neighborhood of the target than far away; there will also be unavoidable boab-hits far outside the area being attacked. However, on the basis of laws of probability, the bonb distribution can be predicted well enough so that the goal of the attack can be achieved with the same or even greater accuracy than for point attack.

## 2. Flight Path of the Bomb

In order to make calculations concerning the bombs thrown from rocket airvraft, we must make some assumptions about the external shape of the projectile. Best suited to the existing conditions is a bullet-shape, with flat base, with a cylindrical tapered portion at the rear, with largest possible ratio of height $h$ ta calibes 1 . With say $\frac{h}{\mathrm{~h}}=8$ we get, from the wellknown gas-dynamic laws for very high Mach numbers, ${ }^{3}$ resistance coefficient of $C_{w}=0.014+1.43$ $A^{2} / V^{2}{ }^{2}$ where in the friction contribution $C_{w r}=0.004$ we neglect the stabilizing surfaces, since they probnbly can only manage to set the projectile spinning about ita axis in the initial part of the flight, before their thin walls are destroyed due to the temperatures developed by friction.

If one assumes that the explosive constitutes $50 \%$ of the homb's weight, then biombs weighing 30 (or 5 or 1) tons have lengths of 11.20 (or 6.16 or 3.60 ) m, and cross-section loadings of 19.5 (or 10.7 , or 6.2 ) tons $/ \mathrm{m}^{2}$. In a practical situation any bomb load could be made up out of these three sizes. The considerable space required for the projectiles will require an adjustment of the bomb-load-and tank-space to the purpose at hand, in the sense that larger fuel loads will be accompanied by decreased bomb loads and vice versa; i.e., the aircraft deseribed earlier is best suited to the first case.

In estimating the path of flight of the bomb we can proceed in the sane way as in determination of the ascendinp or descending path of the aircraft itself. The force picture differs from that for the climbing aircraft, since we have assumed that no aerodynamic lift forces act on the bomb and that the bomb can no longer be kept in a definite orbit plane by a pilot, so that it follows the tendencies to sideways motion due to atmospheric rotation (weather-vane action) and earth rotation (Coriolis.force), and describes a twisted orbit in space. As shown previously, the weather-vane action is caused by the fact that the bomb, as it flies over places of different latitude, continually moves through layers of air of different absolute speed (depending on latitude), and thus is acted on by a cross-wind, which produces horizontal trans. verse forces; these do not act at the center oi mass of the projectile, but behand it, at least. if the projectile has fins. This force as well as the Coriolis force was assumed, in the case of the aircraft, to be eliminated by transverse steering witich develops forces equal and opposite to them. In the case of the bomb there are no controls; as a result of the weather-vane action it will not only drift to the side, but also as a consequence of the resultant torque, it will start to rotate about a vertical axis through the center of mass. The flight path of the bonb
is thus determined by five external forces, of which four act at the center of mass and the fifth acts behind it; weight of the bomb, air resistance, transverse air force, Coriolis force, and d'Alembertian inertial force.

The transverse force is perpendicular to the tangent to the path, points the way the transverse wind is blowing, is proportional to the air resistance and the square of the angle under which the unperturbed air stream meets the synunetry plane of the aircraft. This angle in turn is determined by the strength of the cross-wind, i.e. by the course of the bamb and its velocity relative to the ground, by the nagnitude of the moment of the crosswind about the centex of mase of the bomb, and finally by the slowness with which the bomb responds to the crosswind and its corque, in turning its apex toward the crosswind. This inertia should be as larpe as possible, since the tendency to turn always exists except when the bomb is in the plane of a latitude circle. We can get an idea of the magnitude of this effect by considering the 300 km . long path of fall for a point of release over a pole of the earth. The difference in cross-wind between the point of release and the point of impact is, in this case, the absolute velocity of rotation $22 \mathrm{~m} / \mathrm{sec}$ of the point of impact. Thus the angle af the cross-wind blowing against che projectile is $1 / 2$ degree for a mean forward speed of $3000 \mathrm{~m} / \mathrm{sec}$. For such angles of incidence. in the Newtonian velocity rezion, the transverse force would be equal in magnitude to the resist. ance of the projectile; the projectile will thus carry out the deflection rapidly despite its large monent of inertia, so that the transverse force cannot increase to such magnitude.

The remaining forces, the Coriolis force and the d'Alemhertian inertial force, can be given from our previous analysis. In doing this we should note that the system of forces acting on the bomb is generally a spatial system, i, e. it cannot be balanced by a single inertial force, but rather by a "screw".

If finally the bomb starts to rotate about its axis of symmetry, thein gyroscopic, Magnus-, and Poisson- effects start, which affect the course of the bomb.

In order to obtain the path of the bomb, the force components along the principal directions of the compass and along the vertical must be deterwined and the equations of motion for the three directions in space must be written down. Integration of these would give an exact description of the twisted path in space. This extensive generalization of the well-known 'Fundamental Problen of Exterior Ballistics' is not directy soluble.

To get a preliminary picture of the path of projection, the range, final velocity of the bomb, time of fall, angle of impact, etc., we can use the well-known simple procedure of Poncelet and Didion for stepwise graphical construction of the path of fall. This procedure takes account of the earth curvature, convergence of verticals, and variation of air density and gravity with altitude, and is therefore the best suited of the usual ballistic procedures for the rocket bomb. In the actual calcilation of the bomb path by this procedure, it turns out that the effect of air resistance on the path is extremely small, first. because of the low air density along most of the path, second because of the small cofficient of air resistatice for the slender bulletshaped body. The paths can therefore be very accurately described as Kepler ellipses, and then the range and angle of impact can be determined from these. The actual velocity of impact of the bonbs with the earth will be decreased by a few percent because of the ajr resistance.

The range of the bomb thrown horizontally and falling along a Kepler ellipse is given by
 From these cquations, the relation between the height at release $H$, the velocity at release $V_{0}$, and the range $W$, is plotted in Fig. 87. The duration of fall can be determined by integrating the path lengths or by a step-by-step calculation of the orbit, and gives values corresponding eo those shown in Fig. 88. For large velocities of release, the earth rotation has a noticeable effect on the range. Since the release point is outside the earth's surface, one can insert for the velocity of release the absolute velocity of the bomb at the point of release, and then obtain the absolute length of the range fron Fig. 88; the relative range on the earth can then be calculated in the usual manner.

One thing to be checked is how warm the bomb becomes in falling through the lower layers of air. In the case of the bonber, which has its high speeds in regions of rarefied air, so that it flies at moderate stagnation pressures, it was assumed that equilibrium between heat intake and radiation can be maintained by having a sbcongly radiating skin for sufficientiy low wall tequeratures, or chat critical thermal stresses can be withstood in the trougha of the path by having a skin with sufficient heat capacrtv. During the fall of the bomb through more denst layers of air, the heat transfer per unit area of the bomb surface will increase greatly, but the bomb's fall lasts a much shorter time, so that one may compute using the heat capacity of the shell of the bomb. The stagnation- and also approximately the frictions, temperature can be



Fig. 88: Dependence of duration of throw as a function of infitial altitude and velocity.


Fig. 89: The third aiming phase of bomb release (see p. 134).
estimated from the gas-dynamic energy equation to be $\Delta T=\gamma^{2} / 2600$ if one assumes that for high temperatures of the boundary laver, the vibrational degrees of freedom of the air moleculas are completely excited, but that no dissociation occurs in the boundary layer. The laws poverning the transfer of heat from the heated regions of the boundary layer to the rigid wall are unknown, but one can set up a rough energy balance by using the estimated coefficieft of laminar friction of the boundary layer $C_{f}=0.0003$ to calculate the work of friction per unit surface per sec. as $\boldsymbol{Z} V=G_{f} \cdot q \cdot V=0.5 C_{f} \rho V^{3}$ for $V=6000 \mathrm{~m} / \mathrm{sec}$ at the earth's surface, this qives $5.4 \mathrm{hp} / \mathrm{cm}^{2}$. If one assumes that a third of this heat goes to the wall, then one obtains the conditions of the jet throat of a powder rocket mentioned on page 8; i.e. an iron jacket ouly 2 cni. thick would only begin to melt after 10 sec . as a result of this heating. From mrre recent measurements of heat transfer at high Mach numbers, it appears to increase more slowly than the rate of friction. al work, i.e. it goes like $\rho^{0.5}$, 5 this indicates that the bomb can go through the critical lower atmosphere even without the use of strongly radiating refractory protective layers for the outer covering.

Finally some general considerations are necessary concerning the accuracy of hitting for bomb release from great altitudes at high speed. In gunnery one assumes a diameter of the $50 \%$ circle equal to the range of fire. For the rocket bomber, the range of fire in this sense is the length of the path of fall of the homb. For a mean length of say 600 km , the probable scatter would be 6 km , if one could release from the bomber with the same accuracy as one can fire from a cannon. The actual error will be the resultant of errors in release, which are determined by position-, velocity-, and direction-errors at the point of release, and of davintions during the fall, which are caused mainly by fluctuations in the density and flow of the eir. The navigation of the rochet bomber to the release point is divided into three aiming procedures. The first phase consists in haring the catapult apparatus lie in the direction of the target. if possible; because of its fixed installation deviations up to $90^{\circ}$ may occur. The correction of this error occurs in the second phase right after takeoff, during the motorless flight or right at the start of the propulsion period, when the direction to the target can be set by means of the compess towithin a few minutes of arc. During the glide, the aireraft must be steered very accurately, since systematic effects - rotation of the earth and atmosphere, and accidental influences, such as small asymetries, errors in steering, fluctuations of air density, and movements of the air, continually tend to bring the aifcraft out of its orbit. During the glide above the troposphere, the third phase, on which the accuracy of the bomb hits deperds, is completed. In this one can think of using astronomical methods for pointing; this will be independent of the influence of the weather or the enemy; in the interference-free stationary glide path before release, it permits deternination of position to an accuracy of a few seconds of arc. corresponding to position errors of litcle over 100 m . To determine the apparent horizon under the excellent visibility conditions available, one can sight on three points of the natural horizon; from the depth of the horizon the altitude can be determined. When the apparent horizon has been fixed, the maintenance of the prescribed orbit in space can be assured by choosing one or several stars in the plane perpendicular to the horizon through the position of the target, and following their apparent motion with a theodolite. If the guiding star, during the dynamie flight, stays in the plane of the orbit (i.e. on the vertical cross-hair) the pilot knows that he is in the prescrihed orbit-plane. On the stationary flight path he can determine his absolute velocity from the apparent motion of the star along the vertical cross-hair, and his position and that of the bomb release from the altitude of the star. Whether the target is visible at the instant of bomb release is unimportant. The determination of the point of release involves only the small error in angle measurement, which corresponds to a few hundred meters. Fig, 89 is a picterial presentation of the conditions during this third aiming phase. The errors arising during the fall of the bomb are more signifirant. If among the factors affecting the spread of projectiles: differences in propulsive charge, vibrations of the barrel, meteorological fluctuations, differences in the projectiles and frrors in direction, only the last two are considered and each is piven equal weight, then the spread would be about $2 / 5$ of $1 \%$, i.e. . $6 \%$ (??), which for a range of projection of 600 km , gives a diamecer of 3.6 km for the $50 \%$ circle. To this is added the error in navigation of the aircraft itself, so that one obtains for the probable deviation from the target of a single release, $W_{\mathrm{f}}=3 \mathrm{~km}$, with which we can make approximate calcula. tions.

## 3. Ballistics of Impacts

The process of impact for point release and area release differ fundamentally in having very different velocities and angles of impact. The processes in point bombing are similar to those for ordinary bombing or heavy mortar fire, so that the necessary results can be written down; e.g. in dive bomb attack by the rocket bomber with 30,5 , or 1 ton bombs, for a final diving aped of 500,300 , or $260 \mathrm{~m} / \mathrm{sec}$, the penetrating power of bombs through the earth's crust is 100,30 , or 12 m. , for reinforced concrete the values are $10 \%$ of the above; the corresponding penetrations through armor plate are $200 \mathrm{~cm} .(1.43$ caliber), $60 \mathrm{~cm}(0.86$ caliber) or 25 cm .
(9. כמ calibe:), in ot'ker vords preater than the strength of all known shap's armor.

Enturely anw conditions oreur for the area bont, which has a velocity of impact 10 times as great. The energy of intiact is much ureater than the energy content of the explosives in the bonb. The stecrith of the material of the bombitself will permit it to perietrate a structure, or even to fu through a city with numerons busldings, because of the small angle of impact; it will not permit penetration into the earth. The elfect will thus be similar to that of a mine. The range of the explosion of a definite amount $G[k g]$ of explosive can be estimated as $1 \times \sqrt{k G}$ under the assumption that the surface destroyed is proportional to the explosive charge; here $r$ is the radius of destruction in meters, and $k$ is a constant which gives the degree of destruction; for air pressures of aboul 2 ditou kfom which produce the worst effects or buildings and smash any except speciallv reinforced ane, $k$ is 3 ; for pressures of $5000 \mathrm{~kg} / \mathrm{m}^{2}, \mathrm{k}$ is 12 , for which value sidpht damagy to structures mcurs, walls are overturned, and gables are destroyed; for $k=25$ the safin ifstance of ordinary huildings fram explosive 5 torehouses is reached, and $k=200$ gives the circle at which window panes and, partly, window frames are broken.

These effects are distributed uniformly in all directions from the point of ixpact of a tey thin-walled bonb, and the result of impact of the rigid body of the shock wave moving with greater than sound velocity through the still air. The developnent of such shock waves in frunt of blunt projectiles flying at supersonic speeds is well known. The phenomenon of an explosion wave is quite similar, except that here the excitation comes only to a small extent from the bomb fragments thrown out of the linnb cover, and mostly from the combustion pases of the explosive, which for adiabatic expansion from the pressure and temperature of the explosion to a normal pressure reach radial velocities of $\sqrt{2 \mathrm{DF}_{\mathrm{\prime}}^{\prime}}=3400 \mathrm{~m} / \mathrm{sec}$ for $\mathrm{E}=1400 \mathrm{kcal} / \mathrm{ke} \mathrm{while}$ indiyidual portions of the gas can reach even greater velocities at the expense of other portions. If the explosive cnergy is also shared with the cover of the bomb (representing say 50 the total mass), then the veiocity of the radial explowion wive drops to $2400 \mathrm{~m} / \mathrm{sec}$, a figure which checks well with actually measured velocities of fraptonts. The exploding material of the bomb collides with the surrounding still air at this velocity, and starts the powerful and far-reaching explosion waye in it.

From the nechanies of the explosion prucess we can get a clear picture of the effect of high impact velocity of an areal bomb on the explosive effect. In the following consideration we thall assume an impact velocity of $8000 \mathrm{~m} /$ tece, which we would get if the aireraft descended to the earth's surface at 8000 m 'sec and release.t the projectile at short range. After detonation of the areal bomb on or above the earth's surface the mass of the resultant ball of fire has

 His - tilt atr. The front face of the explosion sphere rollides with the air at a velocity of
 Linus as much encrify content. The antensity of the explosion wave there is 18 times as great as for a bomb exploding at rest. The intensity drops rapidly for the sideward directions, and disappeas completely at the rear.

Since fir area of destruction by an explesive charpe is proportional to the weight of che - : 1 lanive, or more precisely to the energy avalable for the explosion wave, the area of destruc.

 ! racie area boses its mrular shape, ans becmes a drop-like area along the direction of release wata outline can be abolated from the square of the resiltant of impact- and explosive shinckvolocities. The ratio of destroved areas for the some bomb for point release and area release
 ing after areal release is much areater than that of an equal-sized normally-dropped bomb, is fan-shaped and points in the durection of release. Thus the effect of the bomb no longer depends only on the eriergy content of the extrilosive: the kithetic energy of the hombalso produces its fillf effect. The effect of the fragmentation of the bursting bomb-shell increases and distributus itself in the same manner as the intensity of the pxplosion wave. We get the instructive conAnsion for the rocket bonber thit, beraber of $t$, additional enerny of impact, the desired derree of destruction of a given surface can be acromplished by area bombing with auch smaller bomb loads than for point bortitie.

## VI. Types of Attack

## 1. Basic Types of Attack

The type of attack procedure to be used by the rocket bomber in any specific case is determined by the nature of the target and its distance from the home base.

The extraordinary variety of targets is discussed in Section VI-9. There we discuss in greater girdil the basic difference between point and area targets, according to which the types of $\# t t=.$. .th be subdivided into point-attack and area-attack procedures.

The indu, jatual types of point attack follow from the requirement that the bomber fly as siowly as possible over the target, so that it may have rather small residual energy there, If in spite of chis, the bember is to retur: to its takeoff field without a stop-over, then after dropping its bombs, over the target it rast be propelled by its own rocket motor until it has acquired a sufficient speed to, pet fore on the corresponding energy. Thus we arrive at a procedure for point attack involving two propulsions and return hone, which consists essentially in having the bomber, after being catapuit:d at the home base, accelerated only till it acquires enough energy to bring it over the garget. There it releases and turns at the lowest possible speed, then starts its motor with the residual store of fuel, to get up enough energy for the home trip, and lands back at its hone base. Very large quantities of fuel are required for this double propulsion. so that this procedure can be used only for litrited ranges of attack (up to 60\% km ) and Jimited bomb lods. Point attacks over freater distances or with larger bomb loads than in this first procedure are possible if the bonier can land not too far from the target, and take on new fuel.

For the point attack procedure with two driving periads, partial turning and auxiliary point, the bomber is again accelerated after catapult from the home base, until the acquired energy carries it just to the target. Then it releases, turns through the required angla at least possible flight speed, starts its motor with a small residue of fuel on board, to get the small amount of energy which carries it to the auxaliary field not far from the target; it lands there and takes on new fuel. Hith this, it takes off again in normal fashion and returns to the home base; it has the possibility of making further bombing attacks on the way home.

If a point attack is to be carried uf over a larger distance or with very great bomb lood and there is no possible auxiliary lansing place fairly near the target, then rockettechnique, as seen at the present time, gtves no possibility of retrieving the bomber and bringinf it back to its home field. If attack on the target seems more important than the bomber jtself (which has only a relatively small material value), then there is the possibility of sacrificine the bomber after the attack. This procedure of point attack with a single propulsion ieriod and sacrifjer of the bomber is, in principle, applicable to all points on the earth's suri. re. [t is, naturally, to be applied to attacks and targets of very special significance, as far example the surprise destruction of a goverament building and the governing group assembled there, to the killing of a single, specially important enemy person, to sinking large enemy transports or warships, blocking of important avenues of commerce (say canals or straits), and to similar special cases; this is less because of the loss of the aircraft than for the more paluable pitot.

For procedures of attack on an area the need to fly slowly over the target disappears, so thit one has more freedom in carrying out the procedure. The nost ohvious procedure for ares wtach. with single propulsive period and recurn home, consists in the bomber being catapulted fina its hone base, and then driven until it gets sufficient energy to get to the vicinity of the rarget, turn and get back home. The turn path uses up very large amounts of energy, so that this attack procedare remains linited to small distances and bomb loads.

Area attack over great distances is very much simplified, if an auxiliary field existe not to far fron the target, so that the boiber can lano and take on new fuel for the return trip. f:: ilis citse the area attack goes as follows: after release the bomber makes a partial turn thronsth angle lesk than $180^{\circ}$ (this requires smaller energy consumption than for a complete tinn! H:su lies to the auxiliary field on its residual energy. This area attack with single prai ition, partial turn and auxiliary field is aplicable to all distances on the earth; it Hss rere... lasever, that within at most a few theusand km . from the target there is a suitable auxiliur ficld, for landing, and which has a takeoff apparitus. In view of the large number of fessible targets for area atiack, this requirement can be fulfilled only in exceptional cases.


The value of the auxiliary point can vary considerably, not only according to its distance from the target, but also because of the size of the required angle of turn. Since large angles of turn are much more harmful than great distances, an obrions idea is to provide auxiliary points beyond all forsecable targets; e.e. beyond the two population concentrations outside of Europe (North America and S. E. Asia), say on the Marianas in the Pacific Ocean or on the islands of that ocean off the Mexican coast; or to secure a single auxiliary point at the antipodes of the home base, say in New Zealand or on the islands east of it. This auxiliary point at the antipodes can always be reached by straight flight without turning, no matter what point on the earth is attacked. Its distance from che target can be very large. On this discussion is based the method of area attack with single propulsion and auxiliary point at the antipodes. Such a single auxiliary point also has the advantage that it can easily be fully equipped to enable aircraft also to make bombing attachs on their journey back to the home base, and that its islané location can be easily protected against enemy attacks; against the most dangerous attacka by enemy fleet units, this could be done by the rocket bombers.

If such an auxiliary point at the antipodes is not available, afeo attacks over large distances can be carried out by having the bomber, after release, fly a straight course all the - way around the earth till it reaches the home base. This is the procedure for area attack with single propulsion and Circunnavigation of the globe.

Sumarizing: all possible procedures for area attack coincide in what happens up to the bomb release; they differ only in the marner of bringing the bomber home after it releases its bomb load.

## 2. Point Attack with Two Propulsion Periods and Reversal of Path

If we denote the weight of the bomber in the separate phases of this attack procedure by $G_{o}$ for the fully laded bomber weighing 100 tons: $G_{1}$ after consumption of fuel for the outward trif. $G_{2}$ after homb release, and $G_{3}$ the empty weight of 10 tons after consumption of the fuel for the homeward flight, then the identical distances of outward flight and return ars determined by identical ratios $G_{1} / G_{0}$ and $G_{3} / G_{2}$ according to $F i g$ go, while the honh leard $K=G_{1}-G_{2}$. Thus we
 shown in Fig. 82, which also gives the range of this attack procedure, Figs. 91. 92,93 show these ranges of attack, for three exhaust speeds $C=3,4$, and $5000 \mathrm{~m} / \mathrm{sec}$ and with approximate inclusion of the earth's rotation, as contours of equal primissible bomb load, on map of the earth's surface. The pictures show the effectiveness of the rocket bomber in a very persuasive fashion, Despite the unfavorable double propulsion the bozber, for the interaediste exhaust speed, is able to carry out point uttacks within a radius of 2000 km ., which inclucfes all the strong points in turope between Moscow and Madrid, Northern swetien and Tripoli, Jreland and Ankara; these attacks can be carried nut with retrenir urouract ont any no matter how small object on land or sea, with a bomb load of 30 tons, whish brmb, foad will stove in all except specially reinforced structures within 300 m ; penctrate earth worls luts m. thiek, and steel armior 1 meter thick: and then the bomper can return honse withent a sleeruver nitha smaller homb load the same bomber can carry its attacks to over 6000 km ; from fierratiy to Gratral africa, lindugtan, Eastern Siberia, the North polar regions, to the east coast of harth drifrica and over the whale North Atlantic.

 attack with two prapalsion periods thas arpars to lave retrantinary pratical importance, and

 low velacity and altitude, the attack will getherallt the suel a surpirise that puen a warshof on the alert will searcely have sufficient Lin to bel hat the benter, much less to ward of the attack.

## 3. Point Attack with Two Propulsion Periods; Partial Turn and Auxiliary Point

This nethod of point attack differs from that of the preceding section only because the return flight can be shorter than the uutward trif, fromithr home base to the target since the landing is to be made at a suitable auxiliary Jncation other than the home base. Because of the small kinetic energy over the target, the ankle of turn is unimportant; the only important quantity is the distance from the target to the auxiliary point, neasured as a fraction $\underline{k}$ of the distance a from takeoff point to target. With the notation of the preceding snction, one has


Cities of more than

- one million.
$x$ Home base
Fig. 91: Bomb load in tons (i.e, percentage of the initial flight weight) of the Rocket Bomber in the case of point attack - double propulsion with intermediate turn around - and with exhaust velocity $c=3000 \mathrm{~m} / \mathrm{sec}$.


Cities of more than

- one million
$x$ Home base
Fig. 92: Bomb load in tons (i.e. percent of the initial filght weight) of the Rockst Bomber in the case of point attack - double propuision with intermadiate turn around - and with exhaust velocity $c=4000 \mathrm{~m} / \mathrm{sec}$.

- Cities of more than one míllion
$X$ Home base

Fig. 93: Bomb load in tons (1.e. percent of the initial flight weight) of the Rocket Bomber in the case of point attack - double propulsion with intermediate turn around - and with exhaust velocity $c=5000 \mathrm{~m} / \mathrm{sec}$.


Largest circle equidistant from


Clties of more than
© one mililon
Home bese
Secondary base in the
Marianas Islande. Fig. 95: Bomb load of a Rocket Bomber in tons (percent of the take-off weight) In the case of a point attack, dog leg path, second propulsion at the knee, and a secondary base in the Marianas Islands. Exhaugt velocity $c=4000 \mathrm{~m} / \mathrm{sec}$.


Fig. 96: Relationship between attack range a and bomb load B, iee. Laad ratio , in the case of point attack with double propalsion and landing at the antipodal point for $c=3000,4000$ and 5000 m/sec.
two characteristic weight-ratios $G_{1 / f 00}$ and $10 / \mathcal{C}_{2}$ for the outward-bound flight and the flight to the auxiliary point, respectively. thus to each distance of attack a and to the corresponding return distance ka, the required weight-ratios and the possible bomb load $B=G_{1}-G_{2}$ can be gotten from Fig. 80. For $k=0$, we get the curves of Fig. 80; for $k=1$, those of Fig. 82, and for all other values of $k$ we get intermediate values, which for the case of $c=4000 \mathrm{~m} / \mathrm{sec}$, are shown in Fig. 94. Compared to the previous naximum possible range of attack of 5000 km , we now have unlimited ranges of attack up to 20000 km . if an auxiliary point is available sufficiently near near the target. Fig. 95 shows the contours of constant bomb-load in using an auxiliary point in the Marianas - i.e. when a landing is made at the auxiliary point after the atiack. Since a point attack with two propulsions and use of an auriliary point is sensible only for those parts of the earth's surface which are nearer to the auxiliary point than to the home base, the largest circle whose points are equidistant from home base and auxiliary point has been marked on Fig. 95. The bomb lines are limited to the region of the earth's surface beyond this. The possible range of attack in this case includes all of East Asia and a large part of the Hestern Pacific. For $c=3000 \mathrm{~m} / \mathrm{sec}$, the bonb-load curves shrink to small circles around the auxiliary point; for $c=5000 \mathrm{~m} / \mathrm{sec}$, they spread out over practically the whole hemisphere opposite turope. When using an antipodal auxiliary point, $a+k a=20000 \mathrm{~km}$, and the possible bonb loads have the values shown in Fig. 96. In this case bomb loads are possible only for quite large ranges of attack for $c=4000 \mathrm{~m} / \mathrm{sec}$, only beyond $17,800 \mathrm{~km}$, i.e. in a small circle around the antipodal point with a 2200 km . radius. Whereas an antipodai auxiliary point can be important for area attacks, it is of value for point attacks only when the point itself is to be protected (say against attacks by a fleet) by rocket bombers from the hone field.

As an example of point attack with two propulsions and auxiliary point, an atcack on the locks of the Panana Canal and landing at an auxiliary field on the American hest coast will be described briefly. For $C=4000 \mathrm{w} / \mathrm{sec}$, the bomb load is 2 tons; the characteristic numbers for the attacking flight are: Takeoff: time 0 sec, weight 100 tons, velocity $0 \mathrm{~m} / \mathrm{sec}$, altitude 0 km , distance travelled 0 km ; Climb from takeoff irack to northwest; 11 sec , after takeoff; weight 100 tons, velacity $500 \mathrm{~m} / \mathrm{sec}$, altitude 0 km. , distance 3 km . End of the motorless flight: time 36 sec , weight 100 tons, velocity $284 \mathrm{~m} / \mathrm{sec}$, altitude 3.7 km , distance travelled 12 km ; End of Climb Period: time 332 sec, weight 26: tons, velocity $4560 \mathrm{~m} / \mathrm{sec}$, altitude 60 km , distance travelled $5 / 2 \mathrm{~km}$; End of Supersonic descent: 3882 sec , weight 26 tons, velocity $300 \mathrm{~m} / \mathrm{sec}$., altitude 14 km. . distance 9390 km . End of Subsonic Descent: the subsonic descent ends with the start of the diving attack; the final altitude is thus determined by the succeding dive. Since for an attack on the Canal-locks maximun accuracy of hits is more important than high impact velocity, of the bomb, the end of the subsonic descent is chosen as 2 km . altitude. From this we get the other numbers: time 4162 sec , weight 26 tons, velocity $142 \mathrm{~m} / \mathrm{sec}$, distance travelled 9450 km ; End of Dive-attack: the dive-attack goes from 2 km . to about 0.5 km , altitude; the final velocity of the dive is abnat $200 \mathrm{~m} / \mathrm{sec}_{\text {; }}$ then the bombs are released and the aircraft goes off with small loss in velocity, approaching to within negligible distances from the earth's surface. From this we get the values: Time 4172 sec , weight 24 tons, velocity $200 \mathrm{~m} / \mathrm{sec}$, altitude 0 km , diatance travelled 9450 km ;
End of the Second Climb Period: tine 4405 sec , weight 10 tons, velocity $2800 \mathrm{~m} / \mathrm{sec}$, altitude 22 km ., distance travelled 9710 km ;
End of the Second Supersonic Glide-Flight: time 6125 sec , weight 10 tons, velocity $300 \mathrm{w} / \mathrm{sec}$, altitude 20 km ., distance travelled 12550 km ;
End of the Second Subsonic Glide-Flight; time 6785 sec, weight 10 tons, velocity $80 \mathrm{~m} / \mathrm{sec}$, altitude 0 km ., distance travelled 12648 km ;
Landing: time 6810 sec , weight 10 tons, velocity $0 \mathrm{~m} / \mathrm{sec}$, altitude 0 km. . distance travelled 12650 km

## 4. Point Attack with Sacrifice of the Bomber

According to the previous considerations the rocket bomber with moderate exhaust sped cra carry out ettacks against small individual targets up to 6000 km . distance from the home base; if there is an antipodal auxiliary point, the range extends to 2000 km . from thjs point: for arbitrary auxiliary points distributed over the earth's surface, the range is the same distance from each of the points. If however the point attack is to be directed at a target in whose i icinity there is no auxiliary point, there is the possibility that the bomber proceeds exactly as described in the previous section, and lands at a edrresponding point near the target evan thougn the technical installations of an auxillary point dp not exist there. In this case it will no longer be able to take otif from this point under its own power but will not in general be lost if the landing does nat occur in enemy territory If there is no other passibility than landing in enemy territory, then we are left with a last, not to be neglected, way out-point attack with single propulsion and sacrifice of the bonber.

$\times$ Homs base
Fig. 97: Bomb load of a Rocket Bomber in tons, that is percent of the initial weight, in the case of point attack with a single acceloration and sacrifice of the bouber and mith exhaust velocity $c=3000 \mathrm{~m} / \mathrm{sec}$.
Fig. 98: Bomb load of a Rocket Bomber in tons, that is percent of the initial weight,

- Cities of more than
one million
$\times$ Home base
+ Antipodal base point

Since the aircraft gains altitude rapidly after bomb release in a point attack, the pilot can, at the and of this brief climb, parachute from the plane and destroy the empty aircraft to keep it from getting into the hands of the enemy. He will land a few km. away from the point of the impact of his bombs, and be captured. Fron the point of view of performance this procedure represents the limiting case of the point attack procedure deberibed in the previous section, for which $k=0$, i.e. the retum filight diatance is zero. Figs: 97 and 98 show the possible bomb-loads for this method of attack. In ull cases, the attacking range covers the entire surface of the earth; at $c=3000 \mathrm{~m} / \mathrm{sec}$. 0.5 tons of bombs can still be corried to pointa most distant fron the home base; for $c=4000 \mathrm{~m} /$ sea. this figure increases to 8 tons, and for $c=5000 \mathrm{~m} / \mathrm{sec}$. to 17 tons. This procedure is maturally also suited to unpiloted use of the rocket bomber.

## 5. Area Autack with Full Turn

This first procedure for area attack corfesponds to the first point attack procedure described, with the difference that the energy supplied at the start of the flight rust suffice for the outward filight and the entire return trip to the home base; thus large kinetic energy is present over the target, and considerable fractions of this energy are lost in turning. An outline of the entire flight path including the path of the bomb is included as a aketch in Fig. 99. In order to calculate the relation shown between bomb-load B and range of attack a, one can proceed as follows: from IV $3, S_{5}$, the length of the subsonic glide is known. From Fig. 79, we get the value of $V_{w 2}$ for ( $a-\mathbb{S}_{5}^{5}-W$ ); frort Fig. 99 we get $V_{w 1}$. Now the length of the climb, $S_{3}$ nust be estimated"by reading off from Fig. 64 a first valuw of $G / G p$ for an assumed $V_{1}$, and then getting from this a first estimate of $S_{3}$. Then ( $s-S_{3}-w$ ) is given from $S_{5}+\left(a-S_{5},-w\right)$ $-S_{3}$. The initial velocity $V_{l}$ an the supersonic descending path, in order to have velocity $V_{\text {wl }}$ after a length of glide of (a $-\mathrm{S}_{3}-W$ ) can be obtained as deseribed from Fig. 79. This value is to be compared with the estimated value and inproved, if necessary. From Fig. 64 we now get the desired $G$ and $B=G-10$. From Fig. B3, the stationary altitude $H_{2}$ for given $V_{w l}$ and $G$, is known, and finally the range of projection of the bomb, $W_{\text {, }}$ corresponding to $V_{01}$ and $h_{1}$ is read from Fig. 87. The range of attack. ${ }_{n}$, is thus $a=S_{5}+\left(a-S_{5}-W\right)+W$. This calculation contains a few issumptions which should be considered briefly. First, the assumption is made that relatinns between velocity and distance calculated for particular flight paths can be transferred, unaltered, to similar flight paths. Wore important is the assumption that the supersonic descending path, during the flight before bomb release and during the tum, is carried out at the stationary altitude, rather than in the strongly oscillating dynamical flight path. This is necessary tor the decisive third phase of aiming, in order to attain the necessary aiming accuracy for the bomb release and in order to release the bombs during horizontal flight. Stationary altitudea of flight art also necessary for the period of turn in order to set up the aerodynamic turningforces.

This last circumstance is connected with the fact that turning is possible only up to definite velocities of flight below the velocity of points on the earth's surface; for higher velocities, other methods of attack nust be used. For this reason the procedure of area attack with single propulsion and full turn is limited to the ranges (up to 12000 km .) marked in Fig. 99. Inside this space it proves to be extrenely effective despite the very cosity turning process.

Figs. 100 and 101 show the lines of equal bomb-weight dropped at lie target by this area attack procedure, for $C=3000$ and $4000 \mathrm{~m} / \mathrm{sec}$. For the Eormer value of exhaust speed, the domain of attack is bounded by a closed curve which deviates from eircle because of the carth's rotation, and whose periphery touches the North Pole, Newfoundiand, Contral Africa, and Central Asia. For $c=4000 \mathrm{ra} / \mathrm{sec}$, the ring expands zo that now only Australia, the South polar regjons, the South racific and the southern tip of South Anerica do not lie within it. for $C=5000 \mathrm{~m} / \mathrm{sec}$. it would cover the whole surface of the earth.

As an example of area attack with single propulsion and full turn, we use the attack on Diev York at a range of 6500 km . For $\mathrm{e}=4000 \mathrm{~m} / \mathrm{sec}$, the bamb lad is 6 tons, and the detailed attarl. runs as follows: the motor starts to work 36 seconds after the take-off at 1 ik km . distance from the takeoff point, and consumes the total fuel supply of 84 tons is the next 336 sec. At the end of the climb process, the aircraft reaches a velocity of $6370 \mathrm{~m} / \mathrm{ser}$, an altirude of 91 km , a distance of 736 km . from the pornt of take-off, and a weight of 16 tons. lising only its store of potential and kinetic energy, the bonber flies on to the point of bomb release, 5550 km . from the take-off poiat, and 950 km . in front of the target. At this point, which is reached 1150 sec , after take-off, the velocity has decreased to $6000 \mathrm{~m} / \mathrm{sec}$, and the stationary altitude to 50 km . After the bonb release the weight is 10 tons. Then the aircraft goes into a turn ithl in $330^{\prime}$ sec. goes through a turn-spiral 1000 km . in diameter until it has reached the direction for the return flight to the home base. Luring turning, the altitude is greatly decreased in rader to develop
09/2=V OT7BI D8OT t
 000006
00006
00008
$0000 \angle$
00000
00005 $0000 \%$ Attack distance $a$ in km Fig. 99: Relationship between attack range a and bomib load B (i.e. load ratio G/G ) in the case of arad attack with turn around for the axhaust velocities $c=3000,4000$ and $5000 \mathrm{~m} / \mathrm{sec}$.



- Cities of more than
one milison
$x$ Home base
Fig. 101: Bomb Load of a Rocket Bomber in tons (i.e. percent of the take-off weight) in the case of area attack with tium around and an exhaust velocity $c=4000 \mathrm{~m} / \mathrm{sec}$.
the aerodynamic forces necessary for the turn. At the end of the turn path; the velocity is still $3700 \mathrm{~m} /$ sec. and the corresponding stationary altitude is 38 ku . The supersonic glide-path in the direction of the home base goes over 5450 km . in 2600 sec . and ends 100 km . before the home base at an altitude of 20 km . and velacity $300 \mathrm{~m} / \mathrm{sec}$. bubsonic glide and landing are completed in customary fashion. The whole flight lasts 4755 sec.


## 6. Area Attack with Partial Turn and Auxiliary Point

This tyfe of attack corresponds to the point attack with two propulsion periods, partial turn and auxiliary point. Like the latter, the area attack discussed here represents the most general case in its class, and includes all other procedures for area attack as special eases.

The course of an area attack with single propulsion, partial turn and auxiliary point consists essentially of first giving the bomber, durine a single propulsion period, all the power which it requires until landing at the predetermined auxiliary point, having the bomber release its bombs in front of the target, at high altitude and flight velacity, as for area bombing, then carry out a turn at the existing high speed immediately after the bomb release, which takes it into the direction of the auxiliary point at which a landing is contemplated, and finally glides with its residual energy to this auxiliary point and lands there.

The first thing to deternine is the relation between bomb load $B$ and attacknange a. This relation is affected by the distance ba between target, and auxiliary point, the angle of turn $\sigma$ through which the bomber goes after release, and the exhaust speed $c$. Because of the large namber of indepondent variables, the relations are many-sided. For example, Fig. 102 ohows the relation between a and $B$ for a large number of distances of return flight, ka, and for a definite angle of turn, $\sigma^{-}=60^{\circ}$. The procedure of computation which gave these curves was the following: Assumine a definite $c_{1}$ a bomb-load $B$ was chosen. To this there corresponds a mass-ratio $G / \mathrm{G}_{0}=$ $(10+B) / 100$. a Jength $S_{3}$ of cjimb path, and from Fig. 59 a final velocity $V_{1}$. For this maximmin velocity $V_{1}$, one can now choose various ranges of artack a $=S_{3}+S_{4}+$ Hin such a way that before the bomb release, (i $e$. at che end of the outward flight over $S_{1}+S_{2}+S_{3}+S_{4}$ ), a sufficient supersonic speed still exists. For this velocity and various turn-angles $\sigma^{-}$, we ean, whe the aid of Fig. 81, calculate the loss of speed and the distance $\$_{w}$ travelled during the turn, and from the residual velocity finally calculate the " distance of return " from release point to landing-point. By several repetitions of this procedure, and interpolation between the rough $\mathrm{S}_{\mathcal{H}}$ - values found, the ka -curves of Fig. 102 were obtained.

Fig. 103 ngain shows the lines of constant, bomb load for a rocket bomber using this procedure of attack, if an anxiliary point on the west coast of America is used. It should be pointed out that for turn-angle $\sigma^{-}=0$, there is a definite bomb-load with which the bomber reaches the auxiliary point and can release its load at any point enroute without altering the range, provided the release occurs after the motor is turned off. Thus for $\sigma=0$ the range of is anywhere between take-off point and auxiliary point. According $t$, Fig. 103 , this bomb load is $S$ tons for $C=3000 \mathrm{~m} / \mathrm{sec}$. With smaller bonb loads the possible area of attack stretches over all of North Anerica and considerable portions of South America and the Pacific Ocean.

## 7. Area Attack with Antipoial Auxiliary Point


#### Abstract

A special case of the area attack described in the previous section occurs if the auxiliary point is at the antipodes of the home base. In this case the turn-angle is aero for all targets. Thus there are no energy losses due to turning at speeds above that of sound. The relation between bomb-load B and attack-range a can be obtained from the equations of the preceding aection for $\sigma=0$ and $a+k a=k$. For a rectilinear flight with totipl distance $a(1+k)=20000 \mathrm{~km}$, with the present approxinations it does not matter at what place on the glide-flight the bombs are released. Thus, within the 20000 km . range of flight, the range of atteck is arbitrary and indeptendent of bomb load. From Fig. B0, the possible bomb laad isio. 7 tons for $C=3000 \mathrm{~m} / \mathrm{sec}, B=8$ tons for $C=4000 \mathrm{~m} / \mathrm{sec}$, and $B=17$ tons for $C=5000 \mathrm{~m} / \mathrm{sec}$,


## 8. Area Attack with Circumnavigation

another special case of the general nethod of area attack discusaed in section VI 6 - the area attack with single propulsion period and circumavigation - results for $\sigma=0$ and a ka $=40000 \mathrm{~km}$. The relation between bomb-load Band attack-range a can be read off from Fig. 80 ,
09/9 = V Of78. puot



[^5]153


+ Secondary base on American Wost Coast
- Citres of more than
one million
$\mathbf{x}$ Home base

FIg. 103:
Bomb load of a Rocket Bomber in tons (1.0. percent of the initial weight) in the cage of area attack using a secondary base on the American west coast and an exhaust velocity $c=3000 \mathrm{~m} / \mathrm{sec}$.
where the rule again bolds that the place of bonb release has no effect on the range, so that the range of attack, $\theta_{\text {, }}$ is completely arbitrary and independent of the bomb load. From this figure, the largest both load with which circumavigation is still possible is $\mathrm{B}=3$ tons for $C=4000 \mathrm{~m} / \mathrm{sec}, B=12$ tons for $C=5000 \mathrm{~m} / \mathrm{sec}$, while for $C=3000 \mathrm{~m} / \mathrm{sec}$, circumavigation cannot be achieved even withont a bomb load. This method of attack shows most clearly the extreme technical superioxity of the rocket bomber which with a size and empty weight equal to that of a medium military craft can, at moderate exhaust speeds, reach every point on the earth's surface with a bomb load of 3 tons, and flies 40000 km . all around thie earth without an internediate landing:

As an example of an area attack with single propulsion period and circumnavigation, we shall use the attack on the city with a million population most distant from Gemmany - Sidney in Anstralia. In this case the range of attick is 16500 km , the possible bomb load is 3 tons. The flight goes as follows: Take-off and motion after take-off do not differ from the same phases of previous examples. 36 sec . after take-off the motor begins to operate and consumes the 87 tons of fuel on hoard in the next 348 sec . At the end of this climbing process the velocity is $7200 \mathrm{~m} / \mathrm{sec}$., the altitude 101 km ., distance from takeoff-point 815 km , and weight 13 tons. This very high initial speed drops to $300 \mathrm{~m} / \mathrm{sec}$. in the course of the supersonic descent which is 39185 km . long. After a 10000 km . journey, the strongly oscillating descent must be damped sufficiently'so that at the release point, 15400 km . from take-off, it runs smoothly onough at the stationary altitude to enable accurate aimity for the bomb release. At the release point the altitude is 49 km ., the velocity is $6400 \mathrm{~m} / \mathrm{sec}$., and the range of projection of the bombs is 1100 km . After release the bomber starts its supersonic glide with only 10 tons weight, during which the course which was previously in a plane has to be altered slightly in order to lead back to the hone base. There the bomber lands 13060 secouds after take-off, having travelled $40,000 \mathrm{~km}$.

## 9. Evaluation of Procedures for Attack

Procedures of point attack are directed against individual houses railroad stations and tracks, tunael entrances, streets, bridges, dams. single ships. canals, dikes, breakwaters, gas. water-and oil-tarks, munitions depots, magazines, power stations, transformer stations, airdromes, harbors, factories, troop concentrations, etc; they are limited to a radius of several thousand km . around the home base, except for special cases where the bomber is sacrificed or flies on to an auxiliary field, when the range of attack can extend over the whole of the earth's surface.

Procedures of area attack can be directed against the entire earth's surface. The probable scatter of bombs over several kilometers limits them to target areas of this nagnitude, e.g. cities with over a million population, large industries, fleets, etc. If in an area attack, the total energy $Z$ in keal is released against a single target with a probable scatter $W_{r}=2 \mathrm{~km}$., then half the hits lie in a circle of 2 kn . radius; the average density of hits on this unit surface is $\bar{z}=Z / 2 w_{p}^{2} \pi_{2}$, the actual density follows a Gauss error curve $z_{r}=1.348 \bar{\Sigma} e^{-0.6 p 4 r 3 w^{2}}$
has the value 1.4 z at the center, and is $/ 1 /$ of this at the boundary of the area 4 km , in diameter. Fig. 104 shows such a distribution curve of bomb hits over a map of New York. If larger connected surfaces than the unit surface described are to be attacked, several points of the target can be aimed at, so that the individual Gauss error curves partially orerlap samewhat like those of Fig. 105, where the distance between aiming-points was chosen as $W_{r} \sqrt{5 \pi}$ so that the average density over the whole surface is $\tilde{z}_{\text {, }}$ while the local densities are shown as contour lines.

From the appropriate literature the following relations can be gotten between average deasity of destructive energy $\bar{z}$ in $k$ cal $/ \mathrm{km}^{2}$ and the resultant destructive effect $\overline{\mathbf{z}}=7 \times 10^{6}$ kfel. puts industrial installations conipletely out of operation for several days.
(Degree of destristion I)
$\bar{z}=1.4 \times 10^{8}$. $\frac{\mathrm{kcal}}{\mathrm{km} 2}$ destroys cities so that all except specially reinforced buildings collapse, and only cellars and foundations are usable (Degree of destruction II)
$\mathbf{z}=1.4 \times 10^{9} \frac{\mathrm{kcal}}{\mathrm{kmi}}$ destroys cities so that cellars are also smashed in, all people inside the area are killed and only foundation wells remain standing (Degree of destruction IlI) $z=7 \times 10^{9} \frac{\mathrm{kcal}}{\mathrm{kmi}}$ makes cities so flat that their location is no longer discernible against the background. (Jegree of destruction (V).

If the energy content of the bomb at rest is assumed to be $\mathrm{E}_{\mathrm{O}}=700 \frac{\mathrm{kch}}{\mathrm{kg}}$, then with $\mathrm{E}=$


Fig. 1043 Hit distribution in area attack against a target point in the center of Now York.


Fig. 105: Target distribution in the case of attack on very large surfaces and contour lines of equal bomb hitting density in percent of average value


Striking velocity in $\mathrm{m} / \mathrm{sec}$.
Fig. 106: Required bamb hit density to produce first to fourth order destruction as a function of the striking velocity of the boub.


Fig. 107: Number of required sorties to cover the unt area and ity sidrrounding, using various methods of attack, with a total of $3.52 \times 10^{9} \mathrm{kcal}$ destruetive energy, as a function of the attack range $a$.
$700+\mathrm{Av}^{2} / 2 \mathrm{~g}$, the density of bonb hits in tons $/ \mathrm{km}^{2}$, independent of the injact valocity V nf the bomb is obtained from Fig. 106.

In order to destroy, to the $2^{\text {nd }}$ degree, the surface of a city having an area equal to the previous unit surface, one must release against the target $Z=2 \times 12.56 \times 1.4 \times 10^{8}=3.52 \times 10^{9}$ keal of destructive energy, corresponding to $5000-420$ toms of bombs depending on the velocity of impact. Then the concentration, as shown in Fig. 104 , rises to $1.96 \times 10^{8} \frac{\mathrm{k}_{\mathrm{k}} \mathrm{g} \frac{1}{\mathrm{~m}} \text { near the aiming- }}{}$ poines, is $0.98 \times 10^{8} \frac{\mathrm{kchif}}{\mathrm{knt}}$ at the edge of the unit circle, and $1.14 \times 10^{8} \frac{\mathrm{kcgh}}{\mathrm{km}} \mathrm{at} 4 \mathrm{kmu}$. distance from the aiming point.

The number of flights necessary for dropping this amount, $Z$, of destructive energy on the unit surface is 84 if the bomb load is 60 tons and the impact velocity is low, or 420 if we assume the smallest bomb load - 1 ton -, and an impact velocity of $8000 \mathrm{~m} / \mathrm{sec}$. Fig. 107 shows plot of the nomber of attacks necessary against the unit surface, for various procedures of attack, against range of attack, for $c=4000 \mathrm{~m} / \mathrm{sec}$. If the unit surface is to be attacked with the ninimum number of flights, then the procedure of point attiack with double propulsion and full turn is best. If a single unit surface is to be bombed, this superiority is doubled, because then all the energy lies inside the unit circle, and only half as many bombs need to be dropped. For greater ranges of autack up to 8000 km ., the procedure of area attack with single propulsion and full turn is far superior to all other procedures, especially since it does not depend on the use of an auxiliary point. A remarkable thing about the curve for this procedure is that the required number of flights does not increase nono onically with the range of attack, but rather that the decreasing bomb load is completely compensated by the increasing impact energy. Corresponding to the full curve, the number of flights actualiy required when using area attack with full turm or circumnavigation fluctuates between 64 at 1000 km . and 322 at 20000 kn . range of attack. The evaluation of attack procedures shown in Eig. 107 assumes that the total consumption of take-off fuel, fuel for the aircraft, and of bombs, which together represent a constant amount of 133.7 tons per flight, shall be a mininum. Since the bombs are much more valuable per unit weight than the fuel, one can also sex a requiremenc of minimum consumption of bombs: In Fig. 108 several procedures of attack age plotited from this viewpoint we sefffere the poini attack procedures are very inferior whileatatack procedures, which operate with high impact velocities of the bombs, are most favorable, especially, area attack with circumnavigation and - at long ranges-, with full turn, which requires in this regina the least total consumption as well as least consumption of bembs.

Fig. 109 shows an idealized distribution of hits, according to the laws of probability, over a city map of Berlin, where it is assumed that 84 bombs of 60 tons weight are dropped on the aiming point with low impact velocity; the half shown lies in the $50 \%$ circle; about each point of impact a circle of destruction of diancter $61 \& \mathrm{~m}$ tesults in which the energy density 1. $4 \times 10^{8} \mathrm{kcal} / \mathrm{km}^{2}$ required for degree II exists,

Fig. 110 shows the corresponding distribution of hits for 140 releases with $8000 \mathrm{~m} / \mathrm{sec}$. impact velocity and 3 tons weight per release; again the average energy in the unit circle is $1,4 \times 10^{8} \mathrm{kcal} / \mathrm{kp}^{2}$ but the area of destruction for the same energy per release is now dropshaped and includes $180000 \mathrm{~m}^{2}$, as previously derived.

## VI. THE LINE OF DEVELOPMENT OF THE ROCKET BOMBER

The development of the rocket bomber project will follow roughly the sequence of 12 stages outlined below:

## 1. Development of the Combustion Chamber and Jet of the Motor

The main problems in this stage concern the introduction of the solid, liquid, or siready vaporized fuel and the combustion-maintaining material, into a combustion chanber through injection nozzles; then the rapid distribution, mixing, heating and ignition of the fuel, its most complete combustion at more or less constant high pressure to a combustion gas at very high tempersture; then the expansion of these geses in a jet to convert them to beam of mas with as high atreming velocity and as low temperature as possible. The very bigh preasures and temperatures in the combustion space have the consequence thet not only the tubes for the streaming process in the motor, but also the construction of oll walls in contact with the flame, becomes a very serious problem, whose solution as regards choice of materials, methods of cooling, and constructional arrangement, should be a main point of study. Also important are questions of shape and relative size of combustion chamber and jet, choice of most suitable flame pressures, measurement of stream temperature and velocity. arrangements for rapid heating and mixing of the fuel. optical and acoustic phenomena, mixing of the jet with the surmounding air behind the motor, dissocintionand detontion-problems, and numberless others.

## 2. Development of Special Fuels for Racket Motors

In meny respects gidite different requirementis wre to be set for rocket motor fucls then for the fuels of ordinary aircraft motors. In the first place, what counts is the available energy content per unit mass of the copbustion mixture of fuel and, say, oxygen, and not the heating value of the fuel alone Thus combustible materigl which hes a lower heat of combustion than the usual hydrocarbons, but ronsunes much lesa oxygen in burning. can develop a far superior heat output of the mixture. In addition to the heat output of the mixture, other combustion characteristics such as ease of ignition, rate of comblistion, tendency to detonate, degree of dissociation, state of aggregation of the ambustion products, resiction temperatures, ete., are important. One must also consider properties not so directly connected with the combustion, such as procurement and cost, ease of storage in tanks on the aircraft, density, danger, ease of feeding of the fuel, etc. If one enwmeretes the problems of atomic hydrogen ond nitrogen, of nuclear reactions and of takeoff fuels, then one has outlined in broad strokes the scope of fued research which should lead to the development of new and more suiteble rocket fuels.

## 3. Development of the Auxiliary Engines of the Rocket Bomber

Just as with ordinary eircrift motors, the rocket notor requires for its operation a few auriliary engines, of which the nost importent are thase for feeding fuel and cool ant and the associated driving assemblies. These additional installations present some not-too-simple problems, since the feeding rates are very high, of order of magnitude $50-100 \mathrm{H} . \mathrm{P}$. per ton of thrust, and the material to be conveyed can be in oltogether unusual states, soy e liquefied gas, a metallic suspension or even solid or liquid metal - and must be very accurately proportioned as well as being fed against very high pressures. A complication arises becaise the feeding installations must be designed under extreme limitations on weight of construction. In addition to chese arrangements for feeding of fuel ond coolant, ignition systems and in some cases intake and regulator systems require consideration.

## 4. Development of Test Model of Complete Rocket Motor

Even if the development of the previously enumerated most important parts of the motor had been achieved taking account of their interactions, patting them together into e ready-to-fly rocket motor and examining their interplay is still a separate and important step. Only now, on the apperatus ready for filight can a bench study be made of the mutual interactions of the comibustion chsiber, jet, fuel and auxaliary engines, so that by suitable adjustments the best results. are obtained for exhaust speed, reliability of performance and construction weight. These bench tests of the complete racket motor are especially important and thorough because they reproduce the conditions during flight very closely; this is in contrast to the ordinary aircraft motor where these conditions ean be imitated only with difficulty and not aven completely by altitude tests. This is mainly connected with the fact that the rocket motor accomplishes the jet formation, combustion and cooling only with its own fuels, and without use of the surrounding atmosphere, so that the differences of velocity, tempersture ond pressure of surrounding air between the bench test and acturl flight can scarcely alter the conclusions. One of the few places where the rocket motor comes in contact with the surrounding air is the mouth of the jet. There the


Fig. 108: Amount of required bombs in tons in order to cover the unit surface and the eqvironment by various methods of attack with a total of $3.52 \times 10^{\circ} \mathrm{kcal}$ destructive energy as a function of attack distance a.


Fig. 109: Ideal Gaussian hit distribution of 42 hits , each of 60 tons of bombs, with small striking velocity in the unit circle in case of surface release against the target, and with area of destruction for each bomb (for example, bombardment of Berlin).


Fig. 110: Ideal Gaussian hit distribution of 70 hits , each of 3 tons of bombs, with small striking velocity in the unit circle in case of surface release against the target, and with area of destruction for each bomb (example, bombardment of Berlin).
combution gas expands to the pressure of the air behind the aircraft; in flight this pressure is practically zero wherems in the open bench test the jet ean expond only to the preasure of the still air around it. Still the differences are not very great, are largely amenable to cal culation, and are noreover such that the bench test considers the nore unfavorable case. A reaction of the slip-atream and the jet on the airc:aft (in particular on the control surfaces) need not be considered for the rocket bomber, because operation of the notor and subsonic velocities of flight never occur logether. Aside irom the purely developmental and test studies on the model, a whole series of physical researches on the rocket motor can be carried out most odvantigeously during this phase of the development: more careful. investigntion of atomizing of the fuel, the actual behavior of the pressure in the combustion chamber, the tenperature distribution in the combustion gas, the actual streaning speeds, the conditions of heat transfer from the combustion gas to the walls and from them to the coolant, the jet formation and sound emission in the furnace-jet, as well as numerous otner questions. Only when in the course of these studies the motor has proven its compleve relisbility of performance, can one consider inatalling it in an aircraft. The development of this aircraft will lave to be done dodgside that of the motor, so that both may be rearly for flight at the same time.

## 5. Wind-Tuncel and Tow-Tests on Models of the Alr-Yrame

The external shape of the rocket bomber is determined by the requirements that existing axperience on screw-propelled aircraft shall be used as much ss possible, that the special conditions of supersanic flight and rear installetion of the notor must be considered, that the glide-number of che aircraft shall be as good as possible at very high Mach numbers, where the aerodynamic forces are proportional to the square of the velocity and angle of attack. Under these general requirements modela of the rocket bomber can be designed; these cannot be tested in the most interesting region of very high Mach numbers, in the apper part of which actuale chemical changes of the streaning medium are possible becanse as yet such artificial air-blasts are not available; one can perform wind-tunnel tests in the range of Mach numbers from 0.08 to 4. In the range of velocities below that of sound, which is inportant for the landing process, using models of reasonable size one can carry out measurements on the merodynamic forces, and especially on the maximum lift of the aircraft, and its improvement by the use of aerodymamic aids suited to the perticular profile. In this range tannel-tests on stability, vibration conditions and manoeuvrebility of the vehicle are important. In the supersonic donain, serodynanic forces, distribution of mir aressure, stability and manoeuvrability, vibration conditions, air temperatures at the stagnation points, and heat transfer to the aircraft are of interest. A whole series of these tests on very small models can be carried out in a supersonic wind tunnel.

For tests on larger models the takeofi track of the rocket bomber is itself a very convenient towing-track; on it even very large models can be towed up to any desired high speed with the aid of rocket propulsion, and then studied. Such tow tests can be carried ont at reasonable cost on a $15-30 \mathrm{~km}$. long track, with models scaled $1: 10$ uF to $1: 1$ at the original speeds of 800 up to $5000 \mathrm{~km} / \mathrm{hr}$, so that the Mach number is duplicated exactly ond the Reynolds number approximately. Such tests would especially consider the general behavior of the aircraft in the neighborhood of the velocity of aound, $1 . e$. between 800 and $1800 \mathrm{~km} / \mathrm{hr}$, for various shopes, angles of attack, arcangements and tilts of control surfaces, etc.; also the distribution of air pressure and temperature on the surface of the aircraft by means of taps, thermoelements, etc., for all velocities; messurement of the location and magnitude of the forces actang on une aircraft, direct tests of stability by means of a Cardan suspension of the moving model at its center of gravity, studies of the reaction of the jet on these conditions especially on the serodynamic forces and the stability investigation of the elastic properties on elastically-similar models; e.g: as regards vibration of the wings and control surfaces, etc.; takeoff and free-flight characteristics of self-stable or remotely controlled models especially during the passoge of the velocity from vilues below the sound velocity to values above, and others. A special problem in such tow-tests, aside from the track itself, is the relative sliding behavior of lubricated surfaces at sliding speeds up to $5000 \mathrm{~km} / \mathrm{hr}$. For flight at arbitrary speeds in regions of very rare acmosphere, in which the soealled gas-kinetic laws are valid, and where neither tow tests or ordinary wind-tunnel tests are fepsible, an extension of the theory is desirable, as well as its experimental justification, say by extension of the welliknown molecular-beam methods to the present problem,

## 6. Constructional Development of the Vehicla

Once a preliminary decision as to the whole orrangenent and shape of the rocket bombet han been made on the basis of wind-tunnel- and tow-tests, one can conmence the construction of the fuselage, wings, control surfaces, and important installations such as the pilot's cabin, the tank installations and auxiliary equipment. In view of the fixed externol shope, the decisive factor for the construction of the fuselage, wings and control surfages is the fact the air pressures which occur are far greater than those on ordinary aircraft. The air pressures on the cockef bonber will certoiniy be in the neighborhood of $3000 \mathrm{~kg} / \mathrm{m}^{2}$, ond will therefore lead to types
of wing construction like those used in buildings or ships. The problen of the pressurized cabin has already been treated in other connections. The results assembled there can be made use of. There are no prototypes on which to base tank installations for large quantities of fuels like liquefied gases, metallic suspensions, etc., and these must be suited to the special conditions. An especially extensive group of apparatus requiring development is the auxiliary equipanent of the rocket bomber, including velocity-, altitude-, and acceleration-meters, instruments for steering, navigation and siming of bombs, optical instruments and meny ocher apparatuses and instruments connected with the special characteristics of rocket flight.

## 7. Bench Tests on Interaction of Motor and Alr-Frame

These tests represent the last stage before the first flight tests, and should check the gatisfactory operation of the motor (which hes so for only been bench-tested) under the conditions where ell the perts have been ossembled into o vehicle. The exact relation between che driving force and the centers of mass and air pressure of the aircraft should be checked to avoid instabilities caused by incorrect placement of the engine; sud finally during these tests the filot can become accustoned to some of the peculiarities of the new sircraft.

## 8. Development and Test of the Taksoff Arrangement

The several kilometers long takeoff path with its arrangement of rails representit relatively simple enginecring construction which presents no special problems. On the other hand, the takeoff sled. which under a load of several hundred tons must be accelerated in a very short time to $11 / 2$ times the velocity of sound and then slowed down even more rapidly, requires special development. This will be concerned especially with the extraordinarily powerful takeoff rocket, the sliding contacts of the sled and the method of braking. Completely reliable operstion of the takeoff track will hove to be carefully tested by cacapult trials on dead weights comparable to the weight of the aircraft.

## 9. Takeoff and Landing Tests on the Bomber

The takeoff tests begin with small fuel load on a taxi-strip which is as long as possible the practically empty bomber, after very brief operation of its own rocket motor obtains the velocity required to float, after which it rolls on its own landing gear. After a sbort hop it drops down again and carries out its first landing, which does not differ from the later landings after long flights. These takeoff tests should be carried out in such a large space that the landing con occur immediately after the takeoff without turnirg the aireraft, in order not to endanger the aircraft by manoeuvers near the ground when its flight characteristics are still uncertain. If after many such tests, the takeoff and landing characteristics are understood, then by using more fucl the aircraft cen be brought to somewhet higher altitudes, sey a few hundred meters, ana then glided back to earth. Thus the essential flight characteristics at low speed will be determined, the arrangement of the control surfaces will be undertaken, the trimang, stability and manouurability will be checked, and thus the airworthiness of the craft in all flight actitudes at not too high altitudes will be determined and eventually improved. If. during these tests the. aircraft acts reliably for the pilot, then the takeof tests on the track can be repeated, by placing the empty aircraft on the takeoff sled, catepulting it and landing it. If these tests are satisfactory the trials can be extended to hagher flight speeds. This phase of the trials will best begin by tow tests on a very long tow peth, so that the aircraft doesn't get to fly but is only accelerated on the towing path and then immediately slowed down agein. These tow tests on the actual aircraft at gliding speeds of 800 to $1800 \mathrm{~km} / \mathrm{h}$ are to be devoted especielly to the following individual problems: rechecking and refinenent of the results of the tow tests on models concerning pressures, temperatures, aerodynamic forces, stipility, vibrations, effects of the driving jet, etc.; studies of the behavior of instruments, apperatus end jet engines under the influence of high seceleratione ond velocities, development of suitable safety devices, e.g. special septs, getting the pilot accustomed to the new phenomena of high accelerations, high velocities, special engines and new arrengements of the aircraft.

## 10. Flight Tests of the Bomber

Finally one can go over to flight tests in which one catapults the aircraft with on initially small but gradually increasing supply of fuel, and lets the motor operate for longer and longer periods. Thus longer and longer climb paths will be traversed one will soon surpass the velocities attained up tb now by the fastest aircraft, and go up to oltitudes never previously reached. The takeoff speeds also increase at the same time. Now begins the most difficult part of the flight studies, since the flight conditions depart farther and farther from any known at present and checked by experience; completely new seronautical territory must be conquered. As far as can be visualized at present, the flight tests will extend to maintenance of requirements of life in the pilot's space taking account of the high altitudes, accelerations, ond temperetures, the effect of the aerodynamic forces on the wings and controls of the aircraft, testing of the heating of the walls at many points of the aircraft, mipintenance of stability and manoeuvrability beyond the velocity of sound, ete. This part of the flight tests will have to be done very
circunspectly and with stepwise increase in velocity, sanct at very high altitudes, low aix densities and high spaeds, any slight scident can lead to a catastrophe since it in practically impossible to leave the aircraft by parachute. These tests of flight characteristics will be continued up to speeds nat yet reached by projectiles, i.e. about 6 times the velocity of sound. Finally the actual trial flights will be completed with performance testa. They will serve first to determine simple performance data such as takeoff speeds, takeoff distances, landing distances, landing speeds, climhing performance, consumption of fuel, etc., and later to determine the maximam speed, ceiling and range of the rocket bomber. Since these three limiting performance figures are interdependent, they can be determined during the same flight. As the velocity is increased from the values attained during the tests of flight characteristics up to the maximum value, the temperature distribution on all windward surfaces will have to be carefully checked by means of a renote-reading thermometer system, in order to cotch in time any dangerous heating as a result of air friction and stagnation. Too great heating, especially of projecting parts, such as the sharp end of the fuselpge, the sherp wing edge, etc., is most dangerous becsuse the slightest melting or other deformation of these carefully shaped eritical points and their consequent blunt,ing leads to instantaneous enormous increase of the stagnation temperature at those places and then in steadily spreading molten regions, ond would result in immediate burning up of the whole aircraft. If the performance flights are carried out until these expected limiting values of $7000 \mathrm{~m} / \mathrm{sec}$ velocity, 150 k altitude and 40000 km . length of flight are reached, then the flight studies can be considered completed.

## Il. Navigation Tests on the Rocket Bomber

The next important task of development concerns arrangements for navigation, which, after propulsion has ceesed, give the rocket eraft exoct knowledge of its path, enable corrections of its course, and permit an exact calculation of the moment of bomb release. This precision navigation will have to be checked in a very large number of flight tests, since the success of ony attack depends on its accurate and rapid performance.

## 12. Bomb Release Trials

These coistitute the lost phase of the research and development wark on the rocket bomber $r$ and should give a practical verificption of the preliminary theoretical work on the processes orcurring during the fall of the bombs and their contect with the earth. So far as the relation between point of impect ond point of relerse from the eircraft for various heights and speeds of flight is concerned, the oceans of the whole earchoprovide a suitebly large testing areas. The study of impact of the bomb on land will be somewhrt more troublesome, since very large uninhabited areas are required. However a few cests in the Arctic regions uncrodden deserts or in our own possessions wall suffice for this purpose.

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a. .............attacking distance ( $m$, km), sound velocity (m/sec)
a..............critical sound velocity (for example, in nozzle throat)
b................acceleration ( $\mathrm{m} / \mathrm{sec}^{2}$ )
$b_{n}$...................... normal acceleration (m/sec${ }^{2}$ )
$b_{t} \ldots . . . . . . . . t$ tangential acceleration ( $\mathrm{m} / \mathrm{sec}^{2}$ )
c........*....effective exhanst velocity (n/sec)

$C_{t h * * * * * . . . . . t h e o r e t i c a l ~ m a x i m m ~ f l o w ~ v e l o c i t y ~ o f ~ f i r e ~ g a s e s ~(m / s e c) ~}^{\text {fill }}$

CH:..............nrobable velocity of the air molecules before they strike a prall (m/sec)
cp.................probable diffuse rebound velocities of the air nolecules from a wall (m/sec)
ca*.............Iift coefficient ( $C_{L}$ )
$c_{\text {no-...........initial }}$ lift coefficient ( $C_{L_{0}}$ )
camax...........maximulift coefficient

$c_{\text {Fr* }}$...........drag coefficient ( $G_{D}$ )
Cwr-...........frictional component of $C_{D}$, drag coeff.
$c_{\text {f.............surface }}$ drag coefficient
c**...........specific heat at constant volume (kcal/ $\mathrm{kg}^{0}$ )
${ }^{\text {C }}$ Vtrans *......snecific heat at constant volume, poxtion due to molecular translation (kcal/ $\mathrm{kg}^{\circ}$ )
$c_{\text {vrot }}$.........specific hcat at constant volume, portion cue to molecular rotation (kcal/kg)

Cosc*........specific heat at constant volume, portion due to molecular vibration (kcal/lsg ${ }^{\circ}$ )
d. rall strength, caliber, diammer (m)
$d_{n}$..............ciamster of nozzle mouth ( m )
d...............nozzle throat diameter (m)
f...............cro:s-sectional areas ( $\mathrm{m}^{2}$ )
f...............cross-sectional areas of throat of fire nozzle ( $m^{2}$ )
$f_{m}$..............area of surface bounding mouth of nozzle ( $m^{2}$ )
g...............acceleration due to gravity ( $\mathrm{m} / \mathrm{sec}^{2}$ )
h..............length of projectile (m)
$h_{0} . . . . . . . . . .$. smallest thichess of lubricating layer (m)
ipHo...........verticel shock impulse mich in unit tine is trangitted to surface of plate by irpinging air molecules ( $\mathrm{kg} / \mathrm{m}^{2}$ )
$i_{p R} \cdot . . . . . . . . .$. vertical rebound impulse thich in unit time is given by surface of plate to rebounding air molecules ( $\mathrm{kg} / \mathrm{m}^{2}$ )
$i_{r} \cdot \ldots . . . . . . . . . \begin{gathered}\text { impuls } \\ \text { narallel }\end{gathered}$ to wall, of the air molecules thich invinge in unit time on a unit area of the wall ( $\mathrm{kg} / \mathrm{m}^{2}$ )
1...............free path length of molecules ( $m$ )
m..................mass (kgsec²/II)
p..............air pressure, gas pressure, preasure ( $\mathrm{kg} / \mathrm{m}^{2}$ )
$\mathrm{p}_{\mathrm{a}}$..............extermel pressure of static atr ( $\mathrm{kg} / \mathrm{m}^{2}$ )
Po...............initial $\frac{\left(\mathrm{kg} / \mathrm{m}^{2}\right)}{}$ pressure, gas pressure in furnace, stationary gas pressure
$p_{m}$..................outh pressure ( $\mathrm{kg} / \mathrm{m}^{2}$ )
q..................dyamic pressure, heat flow ( $\mathrm{kcal} / \mathrm{m}^{2} \mathrm{sec}$ )
r.................trajectory radius of horizontal turning curves (m)
s...............fight distance ( $\mathrm{m}, \mathrm{km}$ ) (pathlength)
s_..............starting distance (m,kmin)
$s_{2} \ldots . . . . . . .$. length of partial distance after starting (m,km)
$s_{3}, \ldots . . . . . .$. length of accelerated clinb ( $m, \mathrm{~km}$ )
s/4...............length of unaccelerated supersonic flide path ( $m, k \mathrm{k}$ )
sfo.............subsonic elide path Iength ( $\mathrm{m}, \mathrm{kn}$ )
$s_{w} \ldots \ldots . . . .$. length of turning distance ( $\mathrm{m}, \mathrm{km}$ )
t..............time (seconds), wing depth, sliding slipper depth (m)
$t_{f}, \ldots . . . . . .$. hot side temperature of fire wall $\left({ }^{\circ} \mathrm{C}\right)$
$t_{k} \ldots \ldots . . . . . . c o o l$ side temperature of fire wall ( ${ }^{\circ} \mathrm{C}$ )
v...............flight velocity ( $\mathrm{m} / \mathrm{sec}$ )
H...............velocity vector (m/sec)
$v_{a} . . . . . . . . . . . a b s o l u t e$ velocity ( $m / s e c$ )
$v_{0} . \ldots . . . . . .$. initial flight velocity (m/sec)
$V_{e} . \ldots . . . . . .$. velocity of a point on the earth's surface ( $\mathrm{m} / \mathrm{sec}$ )
$v_{\text {wl }}, \ldots . . . .$, , flight velocity at beginning of turning arc (m/sec)
$v_{\text {wR }}$............flight velocity at and of turning arc ( $\mathrm{m} / \mathrm{sec}$ )
w................bomb range (m,kn) (literaily "throwing distance")
$W_{r}$..................probable scattering of bomb trajectories (km)
E..............average density of bomb hits for surface attack (keal/ $\mathrm{km}^{2}$ )
A...............mechanical equivalent of heat ( $1 / 427 \mathrm{kcal} / \mathrm{kg}$ ) , 1 lift ( kg )
B...............bomb load (kg)
C....................
D...............disscciation energy (keal/kg)
E..............reaction heat of fuels, upper mixture coefficient, total energy content ( $\mathrm{kcal} / \mathrm{kg}$ )
$\mathrm{F}_{\mathrm{v}}$................spacial energy concentration (kcal/Liter)
$\mathrm{F}_{\mathrm{R}} \ldots \ldots \ldots . . . \begin{gathered}\text { spbound enargy } \\ \text { surface }\left(\mathrm{kcal} / \mathrm{m}^{2} \mathrm{sec}\right)\end{gathered}$
Fwo...........energy of air molecules remaining in walls after inpact
${ }^{\text {F }}$ A.............energy carried hy air molecules per unit time and unit surface
F...............aerodynamic supporting surface (m ${ }^{2}$ )
$\mathrm{F}_{\mathrm{F}} \ldots . . . . . . . .$. carrying wing surface ( $\mathrm{m}^{2}$ )
$\mathrm{F}_{\mathrm{R}}$................carrying fuselage surface ( $\mathrm{m}^{2}$ )

Go..............initial weight (kg)
$G_{1} \ldots \ldots . . . . . \begin{gathered}\text { weight of bomber after using up fuel required in approach of } \\ \text { targat }\end{gathered}$
G,.............weight of bomber after release of bombs (kg)
$\mathrm{G}_{3}$..............emipty weight of bomber (kg)
$G_{s} . \ldots . . . . . . .$. starting weight (kg)
Goa............initial starting weight (kg)
H..................fight altitude ( $n, \mathrm{~km}$ )
J.................heat content (kcal/kg), impulse (kgsec)
J................heat content in state of rest, state of furnace, initial state (kcal/kg)
$\delta_{\text {m }}$.........................
K..............evaluation number of rocket fuels
M.....................elecular weight
P..............effective thrust, load on one of sliding slippar (kg)
P...............free thrust measurable by dynamoxeter ( $\mathrm{k}_{\mathrm{g}}$ )
Q............... quantity of heat (kcal/kg)
R...............individual gas constant ( $m /{ }^{\circ}$ ), radius of earth ( $m$ )
R.................Reynold's number

T................temperature ( ${ }^{\circ} \mathrm{K}$ ), d'Alembert's inertial force ( kg )
$T_{0}$...............initial temperature, furnace temperature, static temperature ( ${ }^{( }{ }_{K}$ )
$T_{\text {ml }}$..................outh temperature ( ${ }^{\circ} \mathrm{K}$ )
$T_{G}$.............temperature of air molecules before impact with wall ( ${ }^{\circ} \mathrm{K}$ )
Two.............temperature of a wall surface struck by air moleculos ( ${ }^{\circ} \mathrm{K}$ )
$T_{R}$..............temperature of air molecules after rebounding from wall ( ${ }^{\circ} \mathrm{K}$ )
U. ..............internal energy (kcal/kg)
V.................specific gas volume ( $\mathrm{m}^{3} / \mathrm{kg}$ )
$V_{0} . \ldots . . . . . .$. Iurnace volume $\left(\mathrm{m}^{3}\right)$
We.................resistance (kg)
Wh...............resistance measured in wind tunnel (kg)
Z..............total energy thrown at a target (kcal)

$\alpha_{\text {s.............angle of attack of leading edge of a curved surface }}$
$\gamma . . . . . . . . . .$. specific weight ( $\mathrm{kg} / \mathrm{m}^{3}$ )
$\gamma^{\prime} . . . . . . . . . .$. specific gas weight at the critical velocity ( $\mathrm{kg} / \mathrm{m}^{3}$ )
$\gamma_{0}, \ldots . . . . . . .$. apecific gas weight at rest, in the neighborhood of Earth's surface $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$
E...............glide number, numerical excentricity of a Koppler ellipse

E,..............optical absorption coefficient
F..............compass rose angle (i.e. "heading") ( ${ }^{\circ}$ )
?.................riscosity ( $\mathrm{kgsec} / \mathrm{m}^{2}$ )
fa................maximum efficiency, external efficiency
Po.............nozzle officioncy
2:...............internal efficiency
Po......................mace efficiency
Ptherm"........thermal officiency
K...............adiabatic axponents
$\overline{\mathcal{H}} . . . . . . . . . . .$. average adiabatic exponents
入...............thermal conductivity (kcal/mh ${ }^{\circ}$ )
r...............coefficient of friction
P..............density, radius of path, ( $\left.\mathrm{kgsec}^{2} / \mathrm{m}^{4}\right)(\mathrm{m})$
F.......................ess of gas atriking unit aurface area of a plate sach second (kgsec/m3)
Pmo............gas density at the mouth ( $\mathrm{kgsec}{ }^{2} / \mathrm{m}^{4}$ )
Po.............gas density in furnace, initial state, rest state ( $\mathrm{kgsec}{ }^{2} / \mathrm{m}^{4}$ )

r..............tangential stress due to air or gas drag ( $\mathrm{kg} / \mathrm{m}^{2}$ )
Y................path inclination ( ${ }^{\circ}$ )

A.................change in a ṭuantity
Q...................characteristic temperature of vibrational excitation of a gas


[^0]:    Pigure 6; Experimental high pressure liquid $\mathrm{O}_{2}$ pump which has a 6 stage rotary puing. At 15,000 i..P.M. it is pupint $5 \mathrm{kj} / \mathrm{sec}$ of liquid 02 at 150 atmospheres.

[^1]:    The tollering table shows in summary form for the cage of stochionetrie combustion of octane in $\mathrm{O}_{2}$ at 100 atm. pressure, how the important final characteristics of the streaming: mouth velocity $C_{m}$, effective exhaust speed $C_{\text {, }}$ jet, mouth temperature $T_{m}$, and the ratio of kinetic energy to total energy supplied (E), differ for the three types of flow and for different jet end-pressures.

[^2]:    Figure 22; Properties of the metal oil dispersions against metal content at 100 atmos chamber pressure.

[^3]:    Numsrical characteristics for three varicus climbing paths of the Rocket Bomber.

[^4]:    Fig. 69: Absolute climbing path of the Rocket Bomber with $c=3000,4000$ and $5000 \mathrm{~m} / \mathrm{sec}$, without consideration of the Earth's rotation, but with consideration of rotation velocity, of a point on Earth's equator, launch to Rast and launch to Meat.

[^5]:    Fig. 100: Area attack - dog leg course - turning anglo $=60^{\circ}=$ constant. ${ }^{1}$

